# Elasticity of Substitution, Small Enterprises and Economic Growth

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**Abstract**

This paper extends the Lucas (1978) model of firm formation by taking into account a normalised CES function in the production process. In a general equilibrium framework it is proved that there is an inverse relation between the value of the elasticity of substitution and average firm size. This relation is also valid, under fairly general assumptions, in steady state.

If interpreted together with the fact richer countries are characterised by a higher elasticity of substitution this result can explain why the recent literature finds a positive association between the importance of SMEs in an economy and its stage of development but seems to fail in finding causality between the two. They have a common origin: a high value of the elasticity of substitution. The paper also provides a first empirical test of the theory proposed using econometric techniques.

JEL Classification: C65, E13, L11

Keywords: Size distribution of firms, general equilibrium models, CES function

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## 1. Introduction

In a very recent working paper Beck, Demirgüc-Kunt and Levine (2003) present evidence that in a very large cross section of countries at different levels of economic development the importance of Small and Medium Enterprises (SMEs) is correlated with GDP per capita growth. However, they also point out that this relation is not robust to controlling for simultaneity bias: while a large sector of SMEs is a peculiar characteristic of well developed economies there seem to be no evidences that SMEs are an engine of growth. Weeks (2003) claims that in developing countries SMEs importance tends to decline at the very early stages of development but that this trend is reversed when these countries experience growth in their income. He therefore points out a U-shaped pattern.

The result that developed countries are now characterised by a large sector of SMEs is not new in the literature. In the last four decades, the importance of SMEs across the globe has grown both in absolute and relative terms (Acs and Audretsch 1993; Loveman and Sengerberger 1991; inter alia). This phenomenon is extremely surprising from a purely theoretical point of view since the conventional wisdom is that larger enterprises are more capable at exploiting scale economies and therefore that as an economy develops they should gain more and more importance. SMEs were thought to be economically less efficient than their larger counterparts, less innovative and offering lower quality jobs.

The proposed explanations for these phenomenon have been numerous: 1) technological change had reduced the extent of scale economies; 2) Increased globalization had rendered markets more volatile and favoured small enterprises; 3) The changing composition of the labour force may be more conducive to small enterprises due to the higher premium placed on work flexibility; 4) changing consumers' tastes facilitated some niches producers; 5) deregulation lowered entrybarriers; 6) increased importance of innovation in high wage countries fostered entrepreneurship and 7) the switch towards a knowledge based economy promoted firm formation[1].

In this paper we argue that a possible explanation of the increase in the importance of SMEs across the globe is related to the elasticity of substitution. We extend the well known Lucas (1978) model of firm formation in order to analyse the effects of a changing elasticity of substitution on average firm size and we prove that under reasonable conditions an increase in the elasticity of substitution bias the size distribution of firms towards smaller firms. To this aim we use a normalised CES function in order to compare economies that differ only in relation to their factor substitutability.

## 2. A (not so brief!) recap of the Lucas model

Lucas (1978) develops a model which takes into account the (given) distribution across individuals of managerial talent and that predicts the size distribution of firm in relation to this distribution of managerial talent[2].

Those individuals who lie at the top end of the spectrum, i.e. the most talented managers run firms and become entrepreneurs; the others prefer to be employees and work for them. In addition to the static features of the model also a very interesting dynamic property is proved: by assuming Gibrat's law, i.e. firm size and firm growth are independent, average firm size increases as the development process, measured by growth in capital per capita, proceeds. This finding is clearly at odds with the Beck,

[1] See Brock and Evans (1989), Audretsch and Thurik (2001), Audretsch (2002) Pietrobelli *et. al* (2004) *inter alia*.

[2] Kihlstrom and Laffont (1979) construct a theory of firm formation which is very similar to the Lucas model. Individuals are not characterised by differences in managerial talent but they only differ in respect to their aversion to risk. They have to choose between starting a new firm and getting an uncertain return to their entrepreneurial activity or join an existing firm and get a certain wage. In equilibrium more risk averse individuals will be employees of firms run by the less risk averse ones.

Demirgüc-Kunt and Levine (2003) results and with the empirical evidence mentioned in the introduction.

In this section we present a shortened version of the Lucas model, which will be then used to present the effects of a changing elasticity of substitution on the size distribution of firms.

*The basic framework:*

Every individual can either be employed in an existing firm or starts a new firm. If she chooses to start a new firm she can produce y units of output using n units of labour and k units of capital via a function f, assuming that the technology exhibits constant returns we can write:

$f(n,k) = nf(r)$ where r=k/n

However output also depends on a parameter x which represents differences in managerial talent and a function g. The parameter x is drawn from a fixed distribution G:R$^{+}$? [0,1] and affects output so that if agent x manages n units of labour and k units of capital her firm produces $xg[f(k,n)]$ units of output where g is an increasing, strictly concave function satisfying g(0)=0. The concavity assumption is needed to avoid the trivial solution that the individual who lies at the top of the spectrum (the best manager) manages all other individuals.

Assuming that the entire distribution of x is always fully represented a resource allocation is described by two functions n(x) and k(x) that describe the amount of labour and capital managed by agent x. An allocation will require that for some agents who are labourers n(x)=k(x)=0 while n(x)>0, k(x)>0 for those agents who become entrepreneurs.

It is therefore possible to obtain a cut-off point z such as if x>z one individual is an entrepreneur and if x<z an individual is an employee.

An efficient allocation is one which maximises output:

$$\frac{Y}{N} = \int_z^{\infty} xg[f(n(x),k(x))]d\Gamma(x) \qquad (1)$$

subject to the two constraints (2) and (3):

$$1 - \Gamma(z) + \int_z^{\infty} n(x)d\Gamma(x) \le 1 \qquad (2)$$

so that the share of people who are entrepreneurs plus the share of people who are employees is smaller than one and

$$\int_z^{\infty} k(x)d\Gamma(x) \le \frac{K}{N} = R \qquad (3)$$

so that no more than the entire amount of capital is employed in the production process. Assuming that the Lagrangian multipliers associated with the constraints (2) and (3) are the wage rate w and the returns to capital u an efficient allocation will also be a competitive equilibrium. The first order conditions of this maximisation problem are:

$$xg'(f)f_n(n(x),k(x)) = w, \qquad x = z \qquad (4)$$

$$xg'(f)f_k(n(x),k(x)) = u, \qquad x = z \qquad (5)$$

Taken together (4) and (5) imply:

$$\frac{w}{u} = \frac{f(r) - rf'(r)}{f'(r)} \text{ since } f(n,k)=nf(r) \qquad (6)$$

Given r from (6) it is possible to obtain the equilibrium size n(x) of firm x from either (4) or (5). Using (5) we get:

$$xg'[n(x)f(r)]f'(r) = u \qquad (7)$$

equation (7) gives firm size, measured by employment, as an implicit function n(x,w,u) if x = z. The function n(?) is increasing in x and u and decreasing in w[3].

[3] This results also applies to the Kihlstrom and Laffont (1979) model, i.e. the less risk averse is the entrepreneur the larger is the firm she manages, provided that uncertainty affects output and the marginal product of labour in the same way.

The two conditions (4) and (5) can be easily rewritten for the "marginal" manager by substituting z in the equations and calculating in a straightforward manner a unique employment level for the marginal manager n(z,w,u).

With the model as such it is fairly easy to calculate an equilibrium solution if both capital per capita and the distribution of x are fixed. However to analyse the dynamics of the model more structure is needed[4]. Following Lucas (1978) we impose Gibrat law, i.e. firm size and firm growth are independent, it is possible to prove that if Gibrat law holds then the function g has the following form[5]:

$$g(\cdot) = a[nf(r)]^{b} \qquad (8)$$

where a and ß are two constants that stem from the solution of a differential equation implied by Gibrat law. Plugging (8) into (7) we get:

$$n(x,w,u) = \frac{1}{f(r)}\left[\frac{u}{abxf'(r)}\right]^{1/(b-1)} \qquad (9)$$

Then bearing in mind that k(x,w,u)=rn(x,w,u), where r is given by (6), inserting n(x,w,u) (from (9)) and k(x,w,u) into the constrains (2) and (3) yields:

$$\left[\frac{u}{abf'(r)}\right]^{1/(b-1)} \frac{1}{f(r)} L(z) = \Gamma(z) \qquad (10)$$

and

$$r\Gamma(z) = R \qquad (11)$$

where $L(z) = \int_{z}^{\infty} x^{1/(1-b)} d\Gamma(x)$ (12)

It is now possible to express per capita output Y/N as a function of z and R, by inserting the employment solution (9) into (1) we get:

[4] Since the problem is not a concave one.

[5] See Lucas (1978) pp. 514 – 515 for a detailed derivation of this result.

$\frac{Y}{N} = a\left[\frac{a\,b\,f'(r)}{u}\right]^{1/(1-b)} L(z)$, now using (10) and bearing in mind that from (11)

r=R/G(z)

we can write:

$$\frac{Y}{N} = a\,(f(r))^{b}\,(\Gamma(z))^{b}\,(L(z))^{1-b} \qquad (13)$$

which represents per capita output when Gibrat law holds. The equilibrium z will therefore maximise (13). The first order condition for this problem is:

$$\frac{b}{\Gamma}\Gamma'(z)\left[1-\frac{r\,f'(r)}{f}\right]-\frac{1-b}{L}z^{1/(1-b)}\Gamma'(z)=0 \qquad (14)$$

In order for (14) to be satisfied it has to be the case that:

$$(1-b)z^{1/(1-b)} \qquad (15)$$

equates

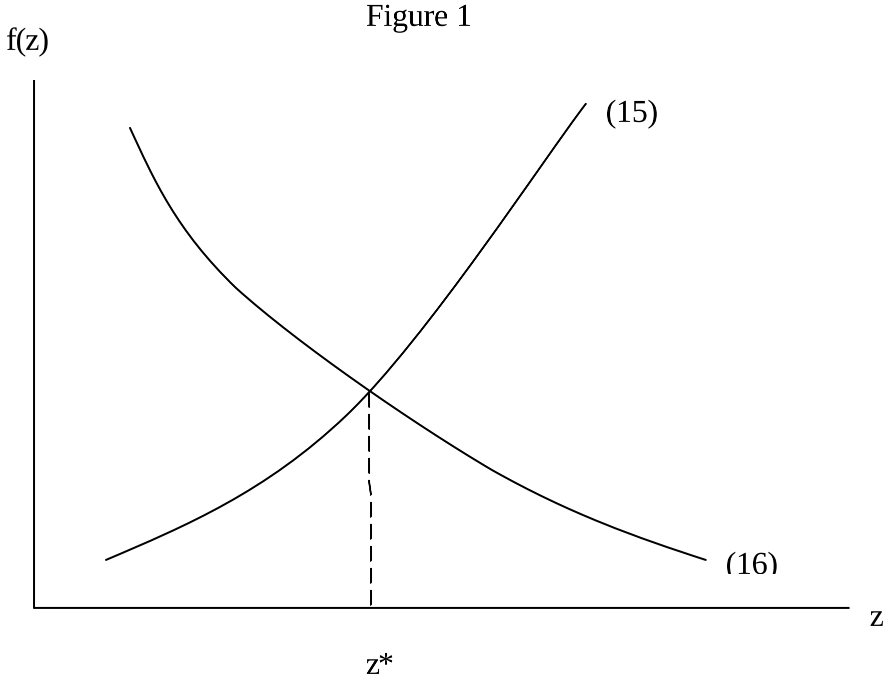
$$b\left[1-\left(r\,f'(r)/f(r)\right)\right]\frac{L(z)}{\Gamma(z)} \qquad (16).$$

The curve (15) passes through the origin and is increasing (and convex). In (16) L/G tends to zero as z tends to infinity and vice versa and assuming that $1-r\frac{f'}{f}$ remains bounded away from 0 and 1 the asymptotic behaviour of (16) is that of L/G. Both function are drawn in Figure 1. In the figure z* represents the equilibrium cut-off point, that is the managerial talent of the marginal manager.

Figure 1 can therefore be used to analyse average firm size in an economy. If z* is very large then the average firm is large as well since there will be more employees and less managers: only individuals with a managerial talent sufficiently large will run firms. On the contrary if z* is low then the average firm will be small since there is little managerial talent required to run a firm.

Lucas (1978) proves that, provided that the elasticity of substitution is smaller than one, as an economy gets richer (i.e. capital per capita increases) average firm size gets larger. This is due to the fact that an increase in per capita capital raises the ratio between the wage rate and the return to managerial activities pushing individuals out of entrepreneurship and into paid work increasing average firm size.

Figure 1




**3. The role of s on the size distribution of firms: using a normalised CES function.**

The purpose of this paper, is to analyse the effects of a changing elasticity of substitution (s) on the size distribution of firms. Since in the Lucas model presented in the previous section it is not specified a precise functional form for the production function it is not possible to pursue this task with the current structure, the model needs to be extended and a precise functional form for the production function taken into account. In this section we assume that the production function has a CES form. More precisely we use a normalised CES function as proposed by Klump and De la

Grandville (2000) and Klump and Preissler (2000). This particular function is extremely helpful in comparing economies that only differ in respect to their factor substitutability.

The normalised CES function is defined by baselines values for the capital stock, the volume of labour, output and the marginal rate of substitution in a point that is common the all functions with different elasticities of substitution. By normalization the efficiency and distribution parameter of the CES function, A and a, are no longer considered constant (except for some important special cases), but become dependent on the elasticity of substitution and the baseline values.

The normalised CES function can be written as[6]:

$$f = A(s)\left\{a(s)r^{y} + 1 - a(s)\right\}^{1/y} \qquad (17)$$

where the elasticity of substitution s and ? are linked by the following relation:

$$y = \frac{s-1}{s}. \qquad (17a)$$

Using the CES function and the two curves (15) and (16) previously obtained it is now possible to analyse the effects of s on the size distribution of firms. We are asking the following question: does average firm size increases or decreases if the elasticity of substitution increases? Since (15) does not depend on s in order to analyse these effects it is sufficient to focus on the function (16). More precisely it is necessary to calculate the first derivative of (16) with respect to s and to study its sign. If the derivative is positive then an increase in the elasticity of substitution shifts (16) to the right so that the equilibrium z-value rises and average firm size increases. On the contrary there is an inverse relation between s and average firm size if the derivative has a negative sign.

The first derivative of (16) with respect to s is:

[6] See the Annex A.1 for the derivation of a normalised CES function.

$$\frac{\partial(16)}{\partial s} = b\,\frac{L(z)}{\Gamma(z)}\frac{\partial\left(1 - r\frac{f'}{f}\right)}{\partial s} \qquad (18)$$

since neither ß nor L/G depend on s .

The key variable influencing average firm size is the profit share $p = r\frac{f'}{f}$. Here clearly emerges that there is an inverse relation between the derivative of the profit share and the derivative of (16): if the derivative of the profit share is positive then (18) is negative and vice versa. Klump and De la Grandville (2000 p.285) proved that the derivative of the profit share with respect to s has the following form[7]:

$$\frac{\partial p}{\partial s} = \frac{1}{s^2}(1-p)p\ln\left(\frac{r}{r°}\right) \qquad (19)$$

Where the symbol ° represents the value of the variable at hand at the point around which the CES function is normalised (this notation will be used through the entire paper). Since (1-p); p and $1/s^2$ are non negative the sign of (19) is determined by the logarithm of r/r°. This logarithm is clearly positive if r/r° is greater than one and negative otherwise. Thus (18) is positive if r < r° and negative if r > r°.

We can therefore conclude that, provided that r > r° an increase in the elasticity of substitution shifts (16) to the left, reduces z* and therefore reduces average firm size (Figure 2). If capital per capita is higher than its baseline value there is an inverse relation between average firm size and factor substitutability. While if it is below r° then the relation is positive. Klump and Preissler (2000) argued that r° should in the general case be below the steady state and that one should focus on results for which this condition holds. One important reason for this assumption comes from the fact that the most popular normalisations of the CES and also of the Cobb-Douglas function implicitly or explicity work with a normalisation r°=1 which means that the capital

[7] For the sake of completeness the derivation is also reported in the Annex A.2.

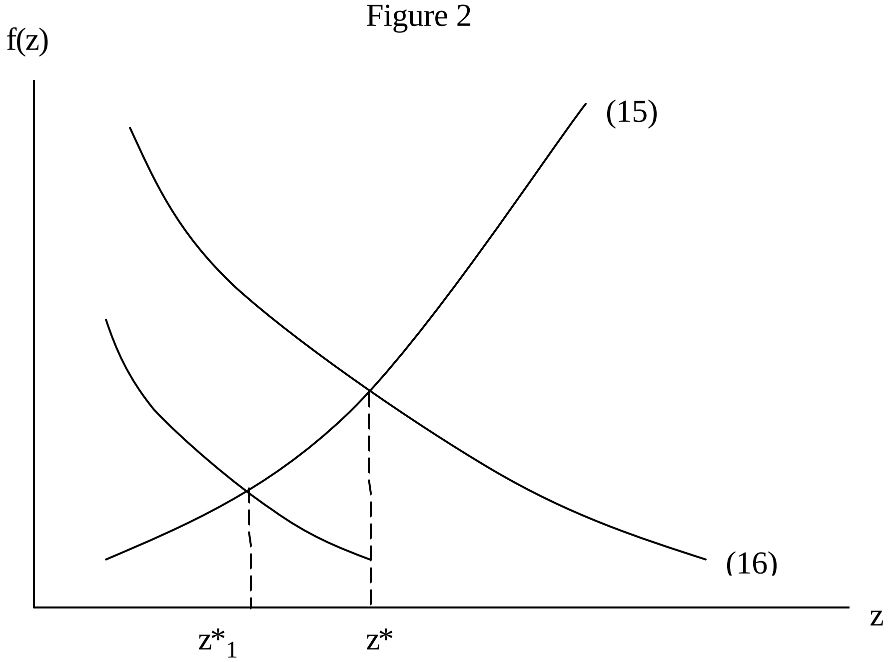
stock is exactly equal to the volume of labour. Developed economies would in general be considered as being more capital intensive then r°=1. There may be cases in very poor developing countries, however, where the assumption r< r°=1 could be justified.

Another fact that is worth noticing is that the profit share depends on s for r different from r°, that is anytime per capita capital differs from its baseline value[8] while it is independent of s if r=r°. The elasticity of substitution has an effect on the profit share and thus on the size distribution of firms only when capital per capita is different from its baseline value.

The logical thinking behind the result that increases in the elasticity of substitution reduce average firm size can be summarised as follows. Recalling that in the model only the "best" managers actually start their own firm the easier it is to substitute factors the easier it is being an entrepreneur, i.e. the managerial talent required to run a firm is lower. An individual with a relatively low endowment of entrepreneurial talent can start his own firm only if it is relatively easy to substitute the factors of production. A very similar argument applies also when individuals differ in respect to their aversion to risk. Increased factor substitutability makes easier for people to become entrepreneurs, i.e. even more risk averse individuals can manage a firm if the production process is more "efficient"[9].

[8] See Klump and De la Grandville (2000) p. 284 for a detailed proof.
[9] De la Grandville (1989 p.479) shows that s may be regarded as a measure of efficiency: the lager its value the larger the benefits that can stem from a change in relative prices.

Figure 2




**4. The steady state implications of a change in s on the size distribution of firms.**

In the previous paragraph we proved that an increase in the elasticity of substitution between labour and capital lowers average firm size under the assumption that the value of per capita capital is larger than r°, i.e. the value around which the CES function is normalised. In this paragraph we are interested in a different question, namely what are the steady state implications for the size distributions of firms for two economies differing only in respect to their elasticity of substitution? In order to provide answer to this question we will be using again the normalised CES function and its peculiar characteristics in steady state. The answer to this question is of particular interest since the result proved by Lucas and the one proved in the previous section are at odds: according to Lucas as the development process goes on small enterprises should become less and less important; this is due to the fact that increases in per capita capital raise the ratio between wages and the return to managerial activity pushing more and more individuals out of entrepreneurship and into paid work. However, our result predicts that as the development process goes on, so that markets function more smoothly, economies become more integrated into world markets and the aggregate elasticity of substitution gets larger, the managerial talent required to run a firm becomes lower so that more and more people start their own firm and average firm size decreases. By analysing the steady state effects of a change in the elasticity of substitution we take into account these two effects at the same time[10] so that it will be possible to single out the net effect on average firm size of the two factors. Klump and De la Grandville (2000) proved that an economy with a higher value of the elasticity of substitution is characterised, in steady state, by a higher value of per capita capital. Since our calculations compute the effects of s on average firm size in steady state they also include the capital deepening due to increases in the elasticity of substitution. Of course if per capita capital is also increasing for exogenous reasons (other than an increase in s) it would be necessary to take into account also these other factors influencing the dynamics of per capita capital.

The analysis of the effects of s on the steady state value of the crucial variable z* starts from the two equations (15) and (16) that we proved to be those that determine the equilibrium cut-off point. Also in this case it is only the function (16) that is influenced by changes in the elasticity of substitution so that it is possible to focus on this equation only. Its form in steady state is the following:

$$(16)_{s.s.} = b\,\frac{L}{\Gamma}\left[1 - r_{s.s.}\,\frac{f'_{s.s.}}{f_{s.s}}\right] = b\,\frac{L}{\Gamma}\left[1 - p_{s.s.}\right] \qquad (20)$$

where the suffix $_{s.s.}$ represents the steady state value of the variable at hand. Again in order to understand the effect of s on the size distribution of firms we have to compute the derivative of (20) with respect to s.

[10] We are going to compute the effects on the profit share of a change in s and then the effects on firm size so that the "Lucas effect" due to capital deepening will be taken into account in our calculations.

$$\frac{\partial(20)}{\partial s} = b\,\frac{L}{\Gamma}\left[1-\frac{\partial p_{s.s.}}{\partial s}\right] \qquad (21)$$

since neither L nor ? depend on s. So that also in steady state there is an inverse relation between the derivative of the profit share and the derivative of (20). The derivative of the steady state profit share with respect to s is[11]:

$$\frac{\partial p_{s.s.}}{ds} = \frac{1}{y}\frac{1}{s^{2}}p_{s.s.}\ln\left(\frac{p_{s.s.}}{p^{\circ}}\right) \qquad (22)$$

It is clear that there are two crucial variables influencing the sign of this function: the first one is ? and the second one the logarithm of the ratio between the steady state profit share and p°. From equation (17a) it is straightforward to prove that ? is positive if s is smaller than one and vice versa[12]. Whilst the ratio can be written as follows[13]:

$$\frac{p_{s.s.}}{p^{\circ}} = \left(\frac{s}{n}\frac{y^{\circ}}{r^{\circ}}\right)^{y} \qquad (23)$$

So that (22) is positive and therefore average firm size is a decreasing function of s in two cases:

1. ?<0 AND $\ln\left(\frac{p^{*}}{p}\right) < 0$ which implies: $\left(\frac{s}{n}\frac{y^{\circ}}{k^{\circ}}\right)^{y} < 1$, but since ?<0 it has to be the case that $\left(\frac{s}{n}\frac{y^{\circ}}{k^{\circ}}\right) > 1$
2. ?>0 AND $\ln\left(\frac{p^{*}}{p}\right) > 0$ which implies: $\left(\frac{s}{n}\frac{y^{\circ}}{k^{\circ}}\right)^{y} > 1$, but since ?>0 it has to be the case that $\left(\frac{s}{n}\frac{y^{\circ}}{k^{\circ}}\right) > 1$

In both cases (if s is larger or smaller than one) in order to have an inverse relation between s itself and average firm size it is required that:

[11] This result has been proved by Klump and De la Grandville (2000). The Annex A.3 reports the proof for completeness.

[12] If the relation between ? and s is drawn on a graph we would obtain an equilateral hyperbole so that if s is larger than one then ? is positive and if s is smaller than one ? is negative.

[13] See the Annex A.4 for the detailed calculations.

$$sy^{\circ} > nk^{\circ} \qquad (24)$$

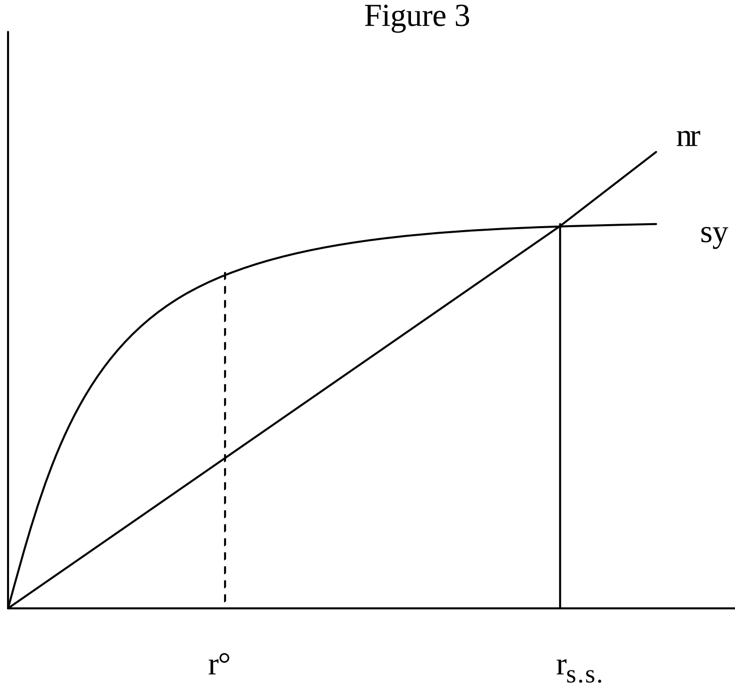
It is now worth recalling that the Lucas (1978) model is equivalent on the production side Solow (1956) growth model[14]. It implies a stable relationship between per capita output and per capita capital which is summarised in an aggregate production function, therefore the dynamic implications for the behaviour of macroeconomic aggregates are the same in the Solow (1956) and in the Lucas (1978) model.

Using the Solow model it is very helpful to understand in what cases condition (24) holds. Figure 3 depicts the two functions sy and nk in the same space as it is usually seen in standard explanations of the path leading to the steady state in the Solow model. It is clear that if we assume that the normalised CES function is standardised around a value of per capita capital which is smaller that the one in steady state then the condition (24) is always fulfilled so that the inverse relation between average firm size and s always holds.

This assumption seems to be a sensible one; a steady state value very close to the origin would require either very low saving rates or very high population growth rates or both conditions together. In such a way then it could be the case that the two functions nr ad sy would cross in a point were both per capita capital and per capita output are very low. This case could be regarded as a poverty trap were it would be necessary to increase the saving rate in order to reach a higher value of output in steady state.

[14] Lucas (1978) p. 518 defines it as a "variant [..] of the Solow-type growth model".

Figure 3



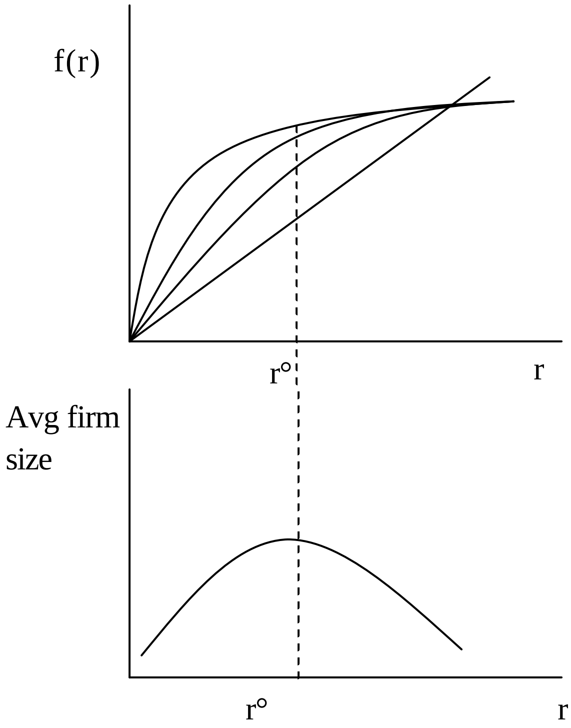

Having proved that, under reasonable conditions, there is an inverse relation between average firm size and the value of s  both in steady state and out of the equilibrium point there are some further considerations that can be made. Both from an empirical (Duffy and Papageorgiou 2000, Miyagiwa and Papageorgiou 2003 and Pereira 2002) and a theoretical (Klump and De la Grandville 2000) point of view a high value of the elasticity of substitution is associated with a higher level of development. This is often related to the good functioning of markets or to integration into international markets. Therefore having a well developed SMEs sector and being a good performer in terms of growth rates have a common origin, namely a high factor substitutability. This very simple consideration can help us to understand (at least partially) the results of the Beck, Demirgüc-Kunt and Levine (2003) working paper that states that although there is evidence that more advanced countries are characterised by a more important SMEs sector there is no causal relationship between SMEs and growth.

The fact that more advanced countries are characterised by a higher elasticity of substitution explains both facts: since the importance of the SMEs sector and growth rates are both related to the elasticity of substitution it is obvious that there is a correlation between these two variables, however at the same time there isn't a causal link between growth and the importance of the SMEs sector. There is (at least one) common origin for the importance of the SMEs sector and performance in terms of growth rates and it is the elasticity of substitution. In the following section we provide a preliminary test of our model by regressing both growth rates and two indicators of SMEs importance on a set of control variables that our model predicts to be crucial in explaining these phenomena.

**5. Some further theoretical considerations.**

It is very useful to analyse together the results presented in Section 3 and 4 in order to understand more deeply what type of dynamics we can expect to observe when we look at an economy which is experiencing an increase in its elasticity of substitution. Figure 4 depicts the relation between s and average firm size for different levels of per capita capital and different curvatures of the CES function. It also summarizes the theoretical results obtained so far. If we assume that the normalisation point r° is on the left of the steady state level of per capita capital then for r<r° we have a positive relation between s and average firm size. At the point where r=r° there is no effect of s on firm size since the elasticity of substitution has no effects on the profit share while for values of r larger than r° an increase in factor substitutability lowers average firm size. In steady state though it is always the case that an economy characterised by higher values of s will be also characterised by a more prominent SMEs sector.

Figure 4




As we stated in Section 2 Lucas proved that an increase in per capita capital favours larger enterprises by raising wages relatively to managerial rents so that moving from low values of r towards its steady state value would favour larger enterprises. What Figure 4 shows is only that this effect is magnified by increases in the elasticity of substitution up to when the condition r<r° holds. If we think of r°=1 this would relate to a situation, where the capital stock is less than the volume of labour. An increase in the investment ratio or an increase in the elasticity of substitution would now increase the capital intensity and also firm size. The two effects will have an opposite sign, however, if capital per capita becomes sufficiently large. This could explain the dynamics of firm size in poor economies as described by Weeks (2003).

We know that in the context of a Solow-type growth model, as the one we are in now, an economy characterised by a higher value of the elasticity of substitution will in general also be characterised by a higher value of per capita capital. In this case the result proved in Section 4 shows that if per capita capital is only increasing because of the elasticity of substitution then the economy will have more SMEs. Therefore in such a situation it is the case that the "s -effect" is larger than the "capital deepening-effect". The elasticity of substitution can be a very powerful instrument for promoting the formation of new firms.

**6. A preliminary test.**

Testing the implications of the model proposed in Sections 3 and 4 is by no means an easy task. An appropriate test would require data on the birth and death of firms and the evolving dynamics of average firm size to be linked with some indicators of the elasticity of substitution and a set of control variables. However such a strategy is not a feasible one for us for a number of reasons: first of all it is difficult to find a panel of firms which includes new firms born and leaves aside the firms that die, in addition to that it is very difficult to find reliable estimates of the elasticity of substitution. These estimates seem to be heavily dependent on the assumption of a particular form for technological change[15], and furthermore they are very often limited to the manufacturing sector. It would therefore be very difficult to estimate the elasticity of substitution and then perform an econometric analysis on SMEs using these estimates. Although we hope to solve these problems in our future research in this study we have therefore been forced to adopt an alternative strategy, namely a cross country analysis, which has to be regarded as a first test and it is therefore not conclusive. We focus only on the steady state implication of our model since the data we gathered do not have a dynamic dimension: we used the data described in Ayyagari, Beck and Demirgüc Kunt (2003) that cover a cross section of countries at different levels of development. These data provide two measures of SMEs importance for the nineties they are sme off and

[15] See Table 1 in Klump, McAdam and Willman (2004)

sme250 which represent the share of formal employment in the SMEs sector according to the official definition of the country at hand and to a common value of 250 employees (where available) respectively. As a measure of the elasticity of substitution we used the Sachs and Warner (1995) indicator for international openness[16], this measure is a dummy variable which takes the value of one if four conditions hold: 1) average tariff rates below 40%; 2) average quota and licensing coverage of imports of less than 40 percent; 3) a black market exchange rate premium that averaged less than 20 percent during the decade of the 1970s and 1980s; and 4) no extreme controls (taxes, quotas, state monopolies) on exports. We averaged this variables through the eighties therefore obtaining an indicator of openness (and of the value of s) which varies between zero and one (sw80). As control variables we used the initial per capita GDP to proxy for the initial development level, the fertility rate to proxy for population growth and investment as a share of GDP to proxy for savings and capital formation. As for fertility and the variable sw80 we used lagged values while for investment an IV estimator has been used[17] to avoid possible endogeneity problems. We then performed a growth regression and a regression on SMEs importance. The models used can therefore be summarised in the following equation:

$$Y = a_0 + a_1 InitialpcGDP + a_2 Fertility + a_3 IVInvestment + a_4 sw80 + e$$

where Y represents either growth rates in the nineties or one of the two indicators of SMEs importance. Table 1 summarizes the results.

The regressions performed seem to confirm the theory presented in our paper: the openness indicator is positively correlated with both growth rates and SMEs

[16] The choice of the measure is clearly an arbitrary one, however with a cross section of countries where none of them can influence international markets alone the degree of openness can be regarded as a good approximation for the elasticity of substitution. For a theoretical paper justifying this choice see Ventura (1997).

[17] The variables in the first stage of the regression are: religious and legal origin variables; a democracy score; indicators of property rights and business environment, inflation rates, dummies for Latin America and Sub-Saharan Africa and an indicator of schooling.

importance. As for the fertility rate and investment the sign of the two coefficients is the predicted one: negative for the first and positive for the second but while the first is not always significant the second is significant at 5% level

**Table 1:** An empirical test of the effects of the elasticity of substitution on growth and SMEs importance

| Dep. Variable | growth90_00 | smeoff | sme250 |
|---|---|---|---|
| N.Obs | 72 | 50 | 37 |
| R-Sqrd | 0.2809 | 0.5323 | 0.6259 |
| F.Stat | 6.54 | 12.80 | 13.39 |
| Prob>F | 0.0002 | 0.0000 | 0.0000 |
| Constant | -.0095918<br>(-0.54) | 45.6601***<br>(3.00) | 46.29721***<br>(2.89) |
| Initialpcgdp | -9.45e-07***<br>(-2.98) | -.0006905**<br>(-2.68) | -.0010469***<br>(-3.23) |
| Fertility | -.000064<br>(-0.03) | -3.085173<br>(-1.56) | -3.929388*<br>(-1.85) |
| Investment | .0015072**<br>(2.12) | 1.352505**<br>(2.27) | 1.507698**<br>(2.38) |
| Sw80 | .0179207**<br>(2.58) | 12.32345**<br>(2.06) | 18.01331**<br>(2.52) |

Note: t-values are in parentheses, ***significant at 1% level,**significant at 5% level, *significant at 10% level. All standard tests have been performed, models are homoscedastic and with normally distributed disturbances.

It looks like the "common origin" between growth rates and SMEs importance is confirmed by this basic test.

## 7. Summary and conclusions.

In this paper we analysed the effects of a changing elasticity of substitution on average firm size in a general equilibrium model and then we argued that s might partially explains some puzzling empirical evidence which finds that SMEs and growth rates are correlated but the latter isn't caused by the former. The basic framework is that of the well known Lucas (1978) model of firm formation in which individuals differ in respect to their managerial ability. However our results can be extended to any model were individuals differ in respect to a parameter which affects the marginal product of labour and total output in the same way: an example of that is the Kihlstrom and Laffont (1979) model where individuals differ in respect to their aversion to risk.

In the Lucas model individuals endowed with a low level of managerial talent become employees in firms run by better talented individuals. A very important dynamic result is that of a positive relation between average firm size and the development process since as per capita capital increases only people with very high endowments of managerial talent become entrepreneurs. This results is driven by the fact that the ratio between the wage rate and the return to entrepreneurship increases with per capita capital. In our paper the Lucas model has been extended by taking into account a specific functional form, namely a normalised CES function. This function, being normalised around some arbitrary values of per capita capital, per capita output and the marginal rate of technical substitution and having its parameters dependent on the elasticity of substitution is a very useful tool and can be used to compare economies that differ only in respect to their factor substitutability. We proved that, under reasonable conditions, an increase in the elasticity of substitution between capital and labour reduces average firm size. Since the elasticity of substitution can be regarded as a measure of efficiency a high value of s makes it more easy for an individual to become an entrepreneur, i.e. if the distribution of managerial talent is fixed over time an increase in the value of s lowers the required talent to run a firm. Therefore in an economy characterised by higher values of s there will be more entrepreneurs and less employees.

It has also been proved that in steady state an economy with a higher s will also be characterised by a more sizeable SMEs sector provided that the steady state level of per capita output is higher than the normalisation point.

Since both from an empirical and a theoretical point of view[18] there is evidence that the level of development and the degree of factor substitutability are positively related we think that this might be a possible explanation for the fact that a study by Beck, Demirgüc-Kunt and Levine (2003) finds that there is correlation but not causation between SMEs importance and growth rates. The relative size of SMEs and growth have a common origin, namely the elasticity of substitution, which exhibits a positive relation with growth rates and an inverse one with average firm size. Thus countries with a high value of s are richer and characterised by a larger SMEs sector but it would be wrong to claim that having a more prominent SMEs sector would have a positive effect on growth.

[18] See Duffy and Papageorgiou (2000), Miyagiwa and Papageorgiou (2003) and Pereira (2002) for the empirical results and Klump and De la Grandville (2000) for the theoretical ones.

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## Annex

*A.1 The normalised CES:*

Starting from a standard CES function $y = A(ar^{y} + (1-a)]^{1/y}$ where $y = \frac{s-1}{s}$ let's take three, arbitrarily chosen, baseline values for per capita capital $r^{\circ}$, per capita production $y^{\circ} = f(r^{\circ})$ and the marginal rate of substitution $m^{\circ} = [f(r^{\circ}) - r^{\circ} f'(r^{\circ})] / f'(r^{\circ})$

We obtain a system of equations in A and a:

$$m^{\circ} = \frac{1-a}{a} r^{\circ 1-y}$$

$$y^{\circ} = A[ar^{\circ y} + (1-a)]^{1/y}$$

and for given values of $r^{\circ}$, $y^{\circ}$ and $m^{\circ}$ we can obtain the two parameters A and a as a function of s:

$$a = \frac{r^{\circ 1-y}}{r^{\circ 1-y} + m^{\circ}} = a(s; r^{\circ}, m^{\circ}) \equiv a(s)$$

$$A = y^{\circ}\left(\frac{r^{\circ 1-y} + m^{\circ}}{r^{\circ} + m^{\circ}}\right)^{1/y} = A(s; r^{\circ}, m^{\circ}, y^{\circ}) \equiv A(s)$$

We obtain the normalised CES function as:

$$f = A(s)\{a(s) r^{y} + 1 - a(s)\}^{1/y} \qquad \text{(A1)}$$

The most popular forms of the normalised CES functions work with a normalisation in which either $r^{\circ} = \left(\frac{k}{n}\right)^{\circ} = 1$ or even k°=n°=1. These normalisation are also implicitly contained in every standard Cobb-Douglas production function (see Klump and Preissler 2000, p. 46, table 2).

*A.2 The profit share and its derivative with respect to s.*

Calculating the profit share using the normalised CES yields:

$$p = \frac{r f'_s(r)}{f_s(r)} = \frac{a r^y}{a r^y + (1-a)} = a\left[\frac{A r}{f_s(r)}\right]^y = \qquad \text{(A2)}$$
$$= \frac{r^y}{r^y + (1-a)/a} = \frac{r^y r^{\circ 1-y}}{r^y r^{\circ 1-y} + m^\circ}$$

While the baseline profit share (for r=r°) can be written:

$$p^\circ = a\left[\frac{A r^\circ}{y^\circ}\right]^y = \frac{r^\circ}{r^\circ + m^\circ} \qquad \text{(A3)}$$

The derivative of the profit share with respect to sigma has therefore the form:

$$\frac{\partial p}{\partial s} = \frac{1}{s^2}\frac{\partial p}{\partial y} = \frac{1}{s^2}\frac{1}{(r^y r^{\circ 1-y} + m^\circ)^2} \times$$
$$\left[r^y r^{\circ 1-y}(r^y r^{\circ 1-y} + m^\circ)(\ln r - \ln r^\circ) - r^y r^{\circ 1-y} r^y r^{\circ 1-y}(\ln r - \ln r^\circ)\right] =$$
$$= \frac{1}{s^2}\frac{r^y r^{\circ 1-y} m^\circ}{(r^y r^{\circ 1-y} + m^\circ)^2}\ln\left(\frac{r}{r^\circ}\right) = \qquad \text{(A4)}$$
$$= \frac{1}{s^2}(1-p)p\ln\left(\frac{r}{r^\circ}\right)$$

which is equation (19) in the text.

*A.3 The steady state profit share and its derivative with respect to s.*

Since in the Solow model the steady state per capita capital can be written as:

$$r_{s.s.} = \left[\frac{1-a}{\left(\frac{n}{sA}\right)^y - a}\right]^{1/y} \qquad \text{(A5)}$$

Then we can compute the steady state profit share as:

$$p_{s.s.} = a\left[\frac{A r_{s.s.}}{f_s(r_{s.s.})}\right]^y = a\left(\frac{sA}{n}\right)^y = \frac{r^{\circ 1-y}}{r^\circ + m^\circ}\left(\frac{s y^\circ}{n}\right)^y \qquad \text{(A6)}$$

Using (A6) and (A3) we can compute the derivative of the steady state profit share with respect to s:

$$\frac{\partial p_{s.s.}}{\partial s}=\frac{1}{s^{2}}p_{s.s.}\left[\ln\left(\frac{sy^{\circ}}{n}\right)-\ln r^{\circ}\right]=\frac{1}{y}\frac{1}{s^{2}}p_{s.s.}\ln\left(\frac{sy^{\circ}}{nr^{\circ}}\right)^{y}=$$
$$=\frac{1}{y}\frac{1}{s^{2}}p_{s.s.}\ln\left[\frac{p_{s.s.}}{a}\left(\frac{y^{\circ}}{Ar^{\circ}}\right)^{y}\right]=\frac{1}{y}\frac{1}{s^{2}}p_{s.s.}\ln\left(\frac{p_{s.s.}}{p^{\circ}}\right) \qquad \text{(A7)}$$

which is equation (22) in the text.

*A.4 The $\frac{p_{s.s.}}{p^{\circ}}$ ratio:*

From equation (A3) and equation (A6) we get:

$$\frac{p_{s.s.}}{p^{\circ}}=\frac{r^{\circ 1-y}}{r^{\circ}+m^{\circ}}\left(\frac{sy^{\circ}}{n}\right)^{y}\frac{r^{\circ}+m^{\circ}}{r^{\circ}}=$$
$$=\frac{r^{\circ 1-y}}{r^{\circ}}\left(\frac{sy^{\circ}}{n}\right)^{y}=r^{\circ -y}\left(\frac{sy^{\circ}}{n}\right)^{y}=\left(\frac{s}{n}\frac{y^{\circ}}{r^{\circ}}\right)^{y} \qquad \text{(A8)}$$

which is equation in the (23) text.