Uncertainty and Risk Aversion: Implication Agriculture

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Abstract

The main aim of this paper is to give the key features for the estimation of production structures using dual approach and to assess risk prevalence in agricultural activity. A supply/demand model based on profit function in which land allocation is set endogenous and conditional on netputs' prices is implemented within a dual approach. Aggregated data from Tunisian cereal crops sector are used and results was characterized, among others, by a low price elasticities of supply. In order to modelize risk non-neutrality into the implemented dual approach, Mean-variance utility function of profit is introduced thereafter to be the objective function for the producer's optimization process rather than profit. Uncertainty was set as consequence of a weather variable variance and randomness of outputs' quantities were set endogenous and conditional on weather variable variance as suggested by Just and Pope (1978) and Coyle (1999). The coefficient of risk aversion was significant and results from the Mean-Variance model shows several salient results relative to both weather and yield variability. The average Risk premium during the sample period was equal to 13.58 percent.

Key words: Agriculture, duality, Mean-variance utility, Risk aversion, Yield uncertainty.

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1. Introduction

Tunisia's agricultural policy has notably been designed to ensure food security for its population. Over the last forty years, the modernization of agriculture and the insulation from external competition has permitted, during the 70's, Tunisia to substantially increase its outputs, yields, and self-sufficiency rates in products considered as being strategic, such as cereals, vegetables, oil and livestock products. But the most troubleshooting fact is that production enhancement was coupled with its high volatility and stagnation of yields in a relatively non adequate level during the 90's. Agricultural productivity, especially in Arid and Semi-Arid Areas remain persistently low and has not kept pace with effective demand. The high level of dependency of agriculture to exogenous conditions such as hazardous and risky natural environment (drought, pests, flooding, insect infestations, disease, etc.) is one of the reasons leading productivity growth below a satisfying level (Mundlak, 1992). That was the main reason leading Tunisian government to implement a large number of administrative regulation mechanisms, the aim being to ensure adequate income levels for farmers and growers, national self-sufficiency for certain products and low prices for staple commodities. These agricultural policy features was likely to entail several "reverse effects". Indeed, as argued by Ben Jemaa (2003), over subsidization of both input purchasing and output prices had led Tunisian large-holders to show inefficient behaviors and a smooth management during the 80's and early 90's.

The context is now changing. Tunisia has started to liberalize agricultural sector after the signature of the GATT agreement, and has taken part in the trade talks on agriculture held under the auspices of the WTO at the end of 1999. It has also engaged in a partnership with the European Union (EU). The agreement of association between Tunisia and the European Union aims at the progressive creation of an Euro-Mediterranean economic space which guarantees a free circulation of goods, capital and services. Three protocols are governing agricultural shutter and have been re-examined in 2001.

The need to design a policy mix that goes with both Tunisia's multilateral and regional commitments and affords sustainable increase agricultural productivity is straightforward. Henceforth, any policy design ought to be based on a sound knowledge of the existing farming systems and decision making behavior of the farm households. Unfortunately, major stresses of livelihoods and their local adaptive and adjustment strategies that have the capacity for becoming the solid basis of sustainable livelihood have not been systematically studied and documented, thus far. farmers' behavior under uncertainty and risk aversion is an important feature which should take a great deal of interest since no work has been undertaken to assess farmers' preferences in Tunisia and even in MENA region except those based on mathematical programming such as those of Bachta (1991) and Hachicha (1993) for Tunisia and studies undertaken by ICARDA[1] in the framework of Mashrek & Maghreb project. Among these characteristics, degree of risk aversion and behavior under uncertainty are corner stone features in modelizing agricultural activity.

A problem arose when thought was given to how risk aversion behavior hypothesis might be tested for. It was realized that any model based, strictly, on the neoclassical theory of the perfectly competitive firm would be inadequate, because its assumption that producers are price taking profit maximizers, operating in exogenously given and deterministic physical and market environments, rules out the possibility that supply response might support risk aversion.

Rather than adopting an entirely *ad hoc* approach, it was decided to critically reassess the standard neoclassical model of production. The value of the theory of the firm derives, as with

[1] International Center for Agricultural Research on Dry Areas.

all economic theories, from the insight it provides into complex economic relationships through providing a highly simplified model of the real world. It is well known that dual approach, although not free of some shortcomings, is consistent with joint output production.
Before these detailed treatments of uncertainty and its incorporation into supply response analysis, a deterministic empirical framework based on duality is estimated, in the first section, within an aggregated model of multi-output production using data from the Tunisian cereal crops sector. Problem of land allocation between the three main cereals crops in Tunisia: hard wheat, tender wheat and barely is implemented by setting land allocation conditional on inputs and outputs (netputs) prices. It will be seen how even though risk neutrality and certainty assumptions ought to lead to "perverse" reactions.
The work carried out on uncertainty and risk aversion is reported in the second part of the paper. In a first time, it is shown how expected utility maximization framework is implemented into a dual approach in order to relax risk neutrality assumption and to make producers' decisions conditional to both weather and output variances. In a second time, an attempt to build up a multi-stochastic-output model for Tunisian cereal crops sector is made using a Mean-Variance utility function. Application of the above methodology is illustrated here using the same data set used for the deterministic model. This permits a comparison with results reported there under the assumption of non-stochastic outputs.

2. Review of the neoclassical theory of production

2.1. The neoclassical theory of production

The standard neoclassical theory of the firm or farm[2] assumes that perfect competition prevails and that economic decision-makers are price-taking profit maximizers. Decisions regarding the level of production, output mix and choice of technique for a firm/farm which transforms m inputs into n outputs are described mathematically in terms of a constrained optimization problem:

$$\max_{X,Y} p = \sum_{i=1}^{n} p_i y_i - \sum_{j=1}^{m} w_j x_j \quad subject\ to\ F(Y,X)=0 \qquad (1)$$

where Y is an (n rows) output vector, X is the (m rows) input vector, $P = p_1 ... p_n$ and $W = w_1 ... w_m$ are the corresponding output and input price vectors, and p represents profits. All inputs are treated as being variable, implying the long-run.
The function $F(Y,X)$, known as the "transformation function", captures the technological constraints governing production. In the single output case, it may reduce to an explicit form, $y = f(X)$ known as the production function[3]. Neither the transformation function nor the production function are entirely arbitrary; in order that the production technologies they describe be "well behaved" - that is, the underlying product-product, product-factor, and factor-factor relationships conform with neoclassical expectations - they must satisfy certain "regularity properties". There is no universally accepted set of regularity properties, since some variation is permitted by economic theory; however, a relatively complete list of

[2] More comprehensive treatments of production theory and duality may be found in a variety of microeconomic texts. See for example: Fuss and McFadden (1978), Varian (1984), Wall and Fisher (1987), and Chambers (1988).

[3] Note that the producer is assumed to be technically efficient. Technical inefficiency may be allowed for by replacing the equality with an inequality: $Y < f(x)$

properties for the production function would include (i) non-regularity, (ii) monotonicity, (iii) twice continuous differentiability, (iv) weak or strict essentiality, and (v) concavity or quasi-concavity[4].

Using the Lagrangian method, the first order conditions for the profit maximization problem given in 1 are :

$$p_i + l \frac{\partial F}{\partial y_i} = 0, \qquad i = 1, \ldots n$$

$$-w_j + l \frac{\partial F}{\partial x_j} = 0, \qquad j = 1, \ldots m \tag{2}$$

$$F(Y, X) = 0$$

where l is the Lagrangian multiplier. Rearrangement and division of the first order conditions yields the well-known result:

$$-\frac{\partial F / \partial x_j}{\partial F / \partial y_i} = \frac{\partial y_i}{\partial x_j} = \frac{w_j}{p_i} \qquad (i = 1, \ldots n; \quad j = 1, \ldots m)$$

Thus, at the optimum, the marginal product for output i with respect to input j is equated to the ratio of those input and output prices.

Solution of the set of simultaneous equations given in (2) yields the optimal output supply and input demand functions[5]:

$$y_i^* = y_i^*(P, W) \qquad i = 1, \ldots n \tag{3}$$

$$x_j^* = x_j^*(P, W) \qquad j = 1, \ldots m \tag{4}$$

Expressions (3) and (4) represent the unconditional or Marshallian output supplies and input demands. Substituting these optimal levels into expression (1) yields the profit function for the firm or farm:[6]

$$p^* = p^*(P, W) = \sum_{i=1}^{n} p_i y_i^*(P, W) - \sum_{j=1}^{m} w_j x_j^*(P, W) \tag{5}$$

The expressions given in (3), (4) and (5) represent the solution to the individual firm or farm's profit maximization problem given the technological and market environment it is assumed to

[4] For discussions of the regularity properties of the production function see Chambers (1988, pp.8-l4) and Gravelle and Rees (1981, pp.l62-67). The regularity properties of multi-product technologies tend to be formulated in terms of set theory; see, for example, Chambers (1988, pp.251-61), Wall and Fisher (1987, pp.6-l0), and Varian (1984, pp.ll-l5).

[5] In addition to the first order conditions given in (2), solution of the profit maximization problem requires that a second order condition and a "total" condition be satisfied as well. See, for example, Gravelle and Rees (1981, pp.220-4), and Beattie and Taylor (1985, p.88). Only the first order conditions, which are needed for the next section on duality, are shown here for the sake of brevity.

[6] $p(P, W)$ is sometimes called the "indirect" profit function to distinguish it from the expression for in the original problem, (1). Note that it corresponds to the maximum, long-run profit obtainable by the firm or farm since all inputs are being treated as variable.

operate in. They provide information on optimal output levels and enterprise selection, choice of technique, and profit level. In addition, depending on the choice of functional form used to specify the production or transformation function and the availability of data for parametric estimation, they may provide information concerning the distribution of income and substitution possibilities between factors of production, the existence of decreasing, increasing or constant returns to scale, technological change, allocative and technical efficiency, and the effects of changes in the market environment. Before going on to review duality theory, it should be noted that the above analysis is based upon a number of assumptions. In particular, one should be noted in preparation for later. Decision makers are assumed to have perfect knowledge of the physical and market environments which they operate in; thus, the possibility of uncertainty and risk aversion is not taken account of.

2.2. Duality

The description of production in terms of a constrained optimization problem constitutes the basis for what is known as the "primal" approach to applied production analysis. There is, however, an alternative approach to the specification and estimation of output supply and input demand relationships, known as the "dual" approach. The primal approach was pioneered by Marshak and Andrews (1944); while the dual approach was developed by Shephard (1953), Diewert (1974) and McFadden (1978). As will be shown below, given appropriate regularity properties, the two approaches provide equivalent representations of a production technology; however, the dual approach has a number of advantages which have made it increasingly popular over recent years.[7]

Just as the underlying transformation or production function must satisfy certain regularity properties, so the profit function given by expression (5) must satisfy certain properties or conditions. The regularity properties of the profit function arise from the profit maximization assumption and the properties of the underlying transformation or production function; they require that the profit function be (i) non-negative, (ii) non-decreasing in the output prices, (iii) non-increasing in the input prices, (iv) convex and continuous in the output and input prices, and (v) positively linear homogeneous in the output and input prices[8]. For any well-behaved transformation of production function satisfying the appropriate regularity properties, there is a profit function satisfying this corresponding set of regularity properties. They are, to all intents and purposes, equivalent representations of the underlying production technology: given a transformation or production function, it is theoretically possible to find the corresponding profit function, and *vice-versa*. These points are elaborated upon below.

In addition to its regularity properties, the profit function has a further property which is extremely useful. Consider the expression for the profit function given in (5); partial differentiation with respect to the price of the k'th output, noting that the y_i^* and x_j^* are functions of p_k yields:

$$\frac{\partial p^*}{\partial p_k} = y_k^* + \sum_{i=1}^{n} p_i \frac{\partial y_i^*}{\partial p_k} + \sum_{j=1}^{m} w_j \frac{\partial x_j^*}{\partial p_k} \tag{6}$$

However, from the first order conditions given in (2), (6) may be rewritten as:

[7] Reviews of duality theory may be found in Wall and Fisher (1987), Chambers (1988), Mundlak (1988) and Coelli (1996).

[8] For further discussion of the regularity properties of the profit function, see Chambers (1988, pp. 124-6), Beattie and Taylor (1985, pp.245-7).

$$\frac{\partial p^*}{\partial p_k} = y_k^* - l\left[\sum_{i=1}^{n} \frac{\partial F}{\partial y_i^*}\frac{\partial y_i^*}{\partial p_k} + \sum_{j=1}^{m} \frac{\partial F}{\partial x_j^*}\frac{\partial x_j^*}{\partial p_k}\right] \quad (7)$$

The expression inside the brackets on the right hand side of (7) is the derivative of $F(Y, X)$ with respect to p_k; but, by definition, $F(X, X)$ is identically equal to zero, thus $\frac{\partial F}{\partial p_k}$ and we obtain:

$$\frac{\partial p^*}{\partial p_k} = y_k^* \quad (8)$$

The above procedure may be repeated for each of the outputs and inputs, demonstrating that the Marshallian output supply and input demand functions given in equations 3 and 4 may be obtained directly from the profit function by partial differentiation:

$$y_i^* = y_i^*(P,W) = \frac{\partial p^*(P,W)}{\partial p_i} \quad (9)$$

$$x_j^* = x_j^*(P,W) = \frac{\partial p^*(P,W)}{\partial w_j} \quad (10)$$

This important and useful property of the profit function, known as Hotelling's Lemma, is an example of a general rule called the Envelope Theorem. Indeed, the same results ought to be obtained using the dual approach based on cost minimization and hence the set of simultaneous outputs supply and inputs demand is derived using Shephard lemma[9]. Elimination of all prices from the set of simultaneous equations in (9) and (10) leads back to the original transformation or production function with which the analysis started above. Solution of the constrained profit maximization problem leads from the transformation function via the output supply and demand functions to the profit function; Hotelling's Lemma means that it is possible to retrace these steps and retrieve the output supply, input demand and transformation functions from the profit function.

The significance of the above analysis is that the information about the underlying production technology contained in the original transformation or production function is also contained in the profit function. As stated earlier, for any well-behaved transformation or production function satisfying the appropriate regularity properties, there is a profit function satisfying a corresponding set of regularity properties.

From a theoretical viewpoint, this means that it is no more arbitrary to start analysis of a particular production technology by choosing a specification for a profit function (or one of the other dual functions such as the cost functions or distance function) than it is to choose a specification for a transformation or production function: there is no loss of generality entailed in choosing the dual rather than the primal approach.

From an empirical viewpoint, the dual approach has a number of advantages over the primal approach. As seen above, the output supply and input demand functions are obtained by

[9] Discussion of duality is restricted here to the profit function, since it is made use of in later chapters. However, there are several other indirect objective functions, both in production theory and in the theory of demand, and the Envelope Theorem can be applied to ail of them. For example, when applied to a cost or expenditure function it yields Shephard's Lemma and when applied to an indirect production function it yields Roy's Identity. For further discussion, sec Colman (1983), Varian (1984, Ch.1), and Chambers (1988, Ch's 2 and 4).

differentiation, whereas in the primal approach they are obtained by solving a constrained optimization problem analytically, a more complicated procedure. Furthermore, prices rather than input quantities are specified as the exogenous variables; this is a more realistic representation, which avoids simultaneous equation bias in single equation estimation. The relevant regularity properties are more easily imposed or tested statistically, and the modeling of multi-product technologies is more flexible.

These proofs of the Law of Supply rely on, among others, one assumption about production which should be made more explicit in the analysis.

This assumption concerns the information available to producers, who are assumed to have perfect knowledge of market conditions, thus excluding any possibility of uncertainty or risk aversion. If uncertainty prevails and producers are risk averse, the deterministic constrained profit maximization model of producer behavior which is the basis of what was exposed *supra* is inappropriate and another model should be adopted. One such possibility is the expected utility maximization model based on von Neumann-Morgenstern utility theory. The extension of the neoclassical model using the von Neumann-Morgenstern approach to allow for the possibility of uncertainty and risk aversion is the aim of the second part of this paper.

Before these detailed treatments of uncertainty and its incorporation into supply response analysis, the next section provides a deterministic empirical framework based on duality.

3. Production technology and land allocation: a profit function approach

3.2. Analytical framework

The most comprehensive analytical approach in estimating output supply functions is to model them in terms of an equation system related to the underlying production technology as stated in expressions (9) and (10).

The total profit function is defined as the sum of all output-specific profit function and joint variable inputs and (quasi) fixed inputs and a non-joint input,

$$p\,(P,W,L,Z) \qquad (11)$$

where P stands for output price vector, W for variable input price vector (labor and fertilizers), L for land allocation vector and Z for fixed inputs vector (capital).

First order conditions gives:

$$\frac{\partial p}{\partial p_i} = y_i^*(P,W,L,Z) \qquad i = 1,2,3 \qquad (12)$$

$$\frac{\partial p}{\partial w_i} = x_i^*(P,W,L,Z) \qquad i = 1,2 \qquad (13)$$

$$\frac{\partial p}{\partial l_i} = \frac{\partial p}{\partial l_j} = 0 \qquad i, j = 1,2,3 \qquad (14)$$

(12) and (13) are (the well known) input demand and output supply equations derived using Hotelling lemma. Equation (14) implies that optimal land allocation between crops is attended when marginal profit relative to land allocation is null, that is there is no gain in changing land allocation. the allocation is done subject to the constraint that the total land area is fixed

in the short run.
Output supply, input demand and land allocation functions can then be estimated jointly with or without profit function. Prominent among the functional forms commonly used to describe a profit or a revenue function is the normalized quadratic (NQ) specification (Villezca-Becerra and Shumway,1994; Guyomard *et al*., 1996) which allows linear equations for the examined products (except for the price used as *numéraire*.).
The preceding analytical framework is used to estimate the supply/demand response for the three principal cereal crops produced in Tunisia, namely hard wheat, tender wheat. and barely (henceforth denoted as crop 1, 2, and 3. respectively) over the last three decades.
The total production profit associated with these crops, given the acreage devoted in the cultivation of each crop is assumed to take the quadratic form,

$$\begin{aligned} p &= b_0 + \sum_{i=1}^{4} a_i w_i + \sum_{i=1}^{2} b_i l_i + \frac{1}{2}\left(\sum_{i=1}^{4}\sum_{j=1}^{4} a_{ij} w_i w_j + \sum_{i=1}^{2}\sum_{j=1}^{2} c_{ij} l_i l_j \right) \\ &+ \sum_{i=1}^{4}\sum_{j=1}^{2} b_{ij} w_i l_j + a_k k + \tfrac{1}{2} a_{5k} k^2 + \sum_{i=1}^{4} a_{i5} w_i k \sum_{j=1}^{2} b_{ik} l_i k + \sum_{i=1}^{4} a_{it} w_i t \\ &+ \sum_{j=1}^{2} b_{it} t + a_t t + \tfrac{1}{2} a_{tt} t^2 \end{aligned} \tag{15}$$

where p denotes total profit divided by the price of barely (crop 3), w_i ($i=1,2,3,4$) denotes netput prices for two inputs: hard wheat and tender wheat and to outputs: labor and fertilizer divided by the price of barely, l_i ($i=1,2$) denotes acreage cultivated for crops 1 and 2. The price of barely is used as *numéraire*. By Hoteling's lemma the supply/demand equations for netputs are given by,

$$\frac{\partial p}{\partial p_i} \equiv y_i = b_i + \sum_{j=1}^{4} a_{ij} w_j + \sum_{j=1}^{2} b_{ij} l_j + a_{ik} k + a_{it} t \quad i, j = 1,2,3,4 \tag{16}$$

where y_i denotes the quantity of netput i.
To these equation described in 21, we add equations of land allocation for crops 1 and 2 (hard wheat and tender wheat). Land allocation equations take the following form,

$$\frac{\partial p}{\partial l_i} = b_i + \sum_{j=1}^{2} c_{ij} l_j + \sum_{j=1}^{4} b_{ij} w_j + b_{ik} k + b_{it} t \equiv 0 \quad i = 1,2 \;\; ; \;\; j = 1,2,3,4 \tag{17}$$

Due to the fact that p is quadratic in its argument, all first derivatives are linear. this specificity allow the following reformulation of land allocation equations,
That is,

$$l_i = -a_i \left(b_i + \underset{i \neq j}{c_{ij} l_j} + \sum_{j=1}^{4} b_{ij} w_j + b_{ik} k + b_{it} t \right) \tag{18}$$

Where $a_i = 1/c_{ii}$, that is the opposite of the inverse of the parameter associated to the quadratic term of land in the profit function.

3.3. Main results

The linear supply/demand and land allocation equations described by Equations (17) and (18) were jointly considered to comprehensively estimate parameters involved in the system. To implement the above specified model, aggregated annual data covering the period 1963-1999 for Tunisian cereal crops sector were used. Time series data on netput prices and quantities was (tediously) compiled essentially from the "Office des Céréales" and the National Statistic Institute (I.N.S). Three broad categories of cereals production factors are compiled using Törnqvist divisia index: capital, Labor and Intermediate consumption. Capital included the flows of material, insurance costs, energy and transport. Labor included hired and family labor. Intermediate consumption included fertilizers, pesticides and herbicides.

To facilitate estimation, all explanatory variables were normalized to unit for 1963. The equations system described in Equations (17) and (18) were estimated jointly using iterative three-stage least squares. The three-stage least squares was required because of the presence of l_i both on the right-hand and the left-hand the equations and also to allow for cross correlation and heteroscedasticity of the errors. The estimation results are presented in the appendix.

Table 1 provides estimates of the crops' supply and input demand responsiveness to own and cross price changes. The own and cross price estimated parameters (which are central here) are statistically significant at least at the 10 percent level except for the labor own price elasticity of demand[10]. Own price parameters are positive, thus complying with economic intuition. In addition, the cross price parameters between the three crops have a negative sign which indicates a substitutability relationship between them. With respect to the *numéraire* equation (barely supply), it should be noted that because of shared parameters with the rest of the model equations, all its price elasticities can be derived from the estimated parameters and they are reported in Tables 1.

Table 1. Own and cross price elasticities (Risk neutrality model)

	Table 1. elasticity of netput (column) relative to the price of (row) :				
	1	**2**	**3**	**4**	**5**
1	1,119	-0,004	-0,357	-0,450	-0,001
2	-0,012	0,152	-0,917	-1,684	-0,026
3	-1,019	-0,917	0,400	-5,544	-0,142
4	0,112	0,840	2,594	-0,889	0,334
5	-0,001	0,065	0,404	1,385	0,076

Elasticities are calculated at sample mean

The sensitivity of supply for the three examined crops to own and cross price changes is further analyzed using elasticities evaluated at the sample mean (Table 1). Since actual cultivated acreage and capital are included among the explanatory variables as (quasi) fixed factors, the computed elasticities represent short-run relationships. Own-price elasticities for all crops production are inelastic except for hard wheat own price elasticity. Inelastic supply ought to be due to technical limitation and to risk aversion. It is known that increasing production (for cereals) increases by the same way its variability and then the productivity effect ought to vanish under the allocation effect (Just and Zilberman, 1986 and Ben Jemaa, 2003). It is worth to sign the discrepancy between the input price elasticities. Demand for labor seems to be in the inelastic range. However, the demand for fertilizers and treatment chemicals are near the unity.

Let's turn to the effects of netputs' prices on the land allocation. Allocation elasticities with respect to netpouts' prices are reported in Table 2. These elasticities are calculated at the

[10] This is due widely to the problems inherent with the compilation and the aggregation. Data used in this framework are the number of actives in the cereals crops sector and number of owners. It is clear that this approximation bring some criticisms but that were the only data available on labor.

sample mean and exhibit with respect to output prices a positive relationship. For tender wheat, own price land allocation elasticity is the smallest. This result comes to corroborate the contention implying that shift in output price, when producer is risk averse, can lead to a null or even ‘perverse’ land allocation as argued by Just and Zilberman (1986) especially if one knows that this cereal crop’s yield is highly volatile. Those effects are endemic for risk adverse producers that proceed to allocate his endowments into production activities according to a judgment between the outcome and the risky nature of the later.

Table 2. effect of netput prices on land allocation
(Risk neutrality model)

Change in land allcation relative to a 1% change in the price of :

Relative to:	Acerage 1	Acerage 2	Acerage 3
1	0,401	0,290	-0,691
2	-0,083	0,033	0,050
3	-0,081	-0,169	0,249
4	-0,181	-0,139	0,320
5	-0,135	0,730	-0,595

Elasticities are calculated at sample mean

The purpose of this section was to implement an application of what was exposed in the section 2 with the introduction of a problem of land allocation. Even risk neutrality and full certainty are implicitly assumed, contention in our results seems to assess for a risk aversion attitude observed in the behaviors of Tunisian cereal crops sector producers. this findings lead to the necessity to take into account the implementation of risk aversion and uncertainty to deep more the analysis and to assess the real magnitude of risk effect on production process. this task will be the purpose of the next section where dealing with risk features are taken into account in the model and lead to more theoretical specifications since some properties of dual form doesn’t hold under such assumptions.

4. Yield Uncertainty and Risk aversion in Duality models: a Mean-Variance approach

Neoclassical production theory assumes that production relationships are known with certainty; and, when this assumption is relaxed the predictions of the standard model can break down. In particular, under uncertainty the Law of Supply need not hold for risk adverse producers. For example, in a simple one output model of pure competition, Baron (1970) showed that when output price is stochastic and the entrepreneur is risk averse it is possible for a firm’s short run supply function to have a negative slope. Similarly, Just and Zilberman (1986), used an allocation model in which production as well as output prices may be stochastic to show that *“any price increase which inherently results in increased variability of returns can cause negative supply response”*.

Both of these results were obtained using extensions of the neoclassical theory of the firm based on von Neumann-Morgenstern (vNM hereafter) utility theory in which the firm is assumed to maximize expected utility of profits (or revenue or any other measure of wealth) rather than simply maximize profits. This approach allows any of the variables concerned, whether endogenous or exogenous, to be treated as random, thereby allowing concepts of uncertainty and attitude towards risk to enter the theoretical framework.

However, although the underlying theory is well developed, little empirical work has been done using the von Neumann-Morgenstern approach (Hallam, Just and Pope, 1982;

Behrmann, 1989; Saha, 1994, Coyle, 1999, among others). This is partly because the duality relationships in deterministic theory, which are so convenient for empirical work, break down under uncertainty. In particular, Pope (1980) has shown that output supply and input demand equations cannot be derived from a profit function by Hotelling's Lemma.

As a result of these difficulties, studies attempting to accommodate risk have tended to adopt *ad hoc* methodologies, often involving single equation estimation of output supply or input demand in which the conventional explanatory variables are supplemented by "risk variables". These variables are usually some measure of the variability of prices or yields, such as a moving average squared deviation or simple range (Brennan, 1982), although the issue of which type of measure is most appropriate is still unresolved, reflecting the problems inherent in the fact that much of the uncertainty facing individual farmers and their attitudes to such uncertainty are not directly observable.

An important theoretical point on which the stress has to be putted is the relationship between risk aversion and degree of efficiency. Attempts to assess both risk aversion and technical (and allocative) efficiency are made by several authors (see for example Horrace and Schmidt, 2000; Fraser and Horrace, 2003; and Kumbhakar, 2004). The consideration of inefficiency can be implemented by a stochastic production function rather than deterministic frontier but it is well accepted that risk aversion holds even under inefficiency. Moreover, the purpose here is basically to assess risk in the production process, in other words, to estimate the role of the risk in deterministic production shifts caused by decision making by farmers in term of their endowment allocation decision and therefore output supply.

In the next part of this paper, those aspects of vNM utility theory which will be used in this analysis, including the Arrow-Pratt measure of risk aversion, the certainty equivalent concept and the constant risk aversion utility function, are summarized; then, shows how the dual approach to applied production analysis can be extended to cover production under uncertainty. A method for modeling a multi output technology where uncertainty enters via random yields is outlined. A model of Tunisian cereal crops production using this approach for is implemented extending neoclassical production theory and duality to include uncertainty and risk aversion. Some concluding comments are made thereafter.

4.1. Uncertainty and risk: von Neumann-Morgenstern utility theory

In deterministic production theory, the firm is assumed to maximize profit. In von Neumann-Morgenstern (vNM) utility theory, it is assumed to maximize its expected utility; where utility is a smooth, monotonically increasing, continuously differentiable function of profit, income or any other measure of wealth. Uncertainty may enter via stochastic production (input and output quantities or yields) or stochastic prices.

If wealth, W, is treated as a random variable with probability density function, $f(W)$, and the firm's utility of profit is $U(W)$, then the expected utility of the firm is,

$$E\left[U(W)\right] = \int_0^\infty U(W) f(W) dW$$

In the vNM utility theory, the objective of the firm is the maximization of $E\left[U(W)\right]$.

Some writers distinguish between uncertainty and risk. For example, Roumasset (1979) has described uncertainty as "*a state of mind in which the individual perceives alternative outcomes to a particular action. Risk, on the other hand, has to do with a degree of uncertainty in a given situation*." However, no distinction is made between the two concepts in expected utility maximization models, where they simply imply that some variables in the

objective function are random.
In the expected utility framework, attitude towards risk is defined in terms of the curvature of the utility function, $U(W)$. If the utility function is concave, viz. $U''(W) < 0$ where $''$ denotes the second total derivative, the decision maker is said to be risk averse. If the utility function is linear, viz.. $U''(W) = 0$, the decision maker is said to be risk neutral. And if the utility function is convex, i.e. $U''(W) > 0$, the decision maker is said to be risk loving or risk taking.
$U''(W)$ is not, however, suitable as a measure of the attitude towards risk, since the vNM utility function is unique only up to a linear transformation. The Arrow-Pratt measure of absolute risk aversion (Pratt, 1964) overcomes this problem by dividing $U''(W)$ by the marginal utility, $U'(W)$, where $'$ denotes the first total derivative, which must be positive, thus:

$$a = -\frac{U''(W)}{U'(W)} = \frac{d \ln U'(W)}{dW}$$

a is not dimensionless since it depends on the units in which wealth is measured[11]. Attitude towards risk can now be characterized as follows:
$a > 0$ implies producers are risk averse,
$a < 0$ implies producers are risk taking,
$a = 0$ implies producers are risk neutral.
The more risk averse the decision-maker is, the larger a is, and the more pronounced the curvature of her or his utility function. Finally, a is unchanged by an arbitrary linear transformation of the utility function. As is apparent from its definition, absolute risk aversion is useful for comparing the attitude of an agent towards a given gamble at different levels of wealth. It seems natural to postulate that agents will become less averse to a given gamble as their wealth increases. This is the notion of decreasing absolute risk aversion (DARA), viz. a is a decreasing function of W (when a is merely nonincreasing in W, the notion is labeled nonincreasing absolute risk aversion (NIARA)).
Sometimes, however, it is interesting to inquire about the attitude of risk-averse decision makers towards gambles that are expressed as a fraction of their wealth. This type of risk preference is captured by the coefficient of relative risk aversion $a_R \equiv Wa$. Unlike the case of absolute risk aversion, there are no compelling *a priori* reasons for any particular behavior of a_R with respect to W. An assumption that is sometimes invoked is that of nonincreasing relative risk aversion (NIRRA), implying that an agent should not become more averse to a gamble expressed as a fixed percentage of her wealth as the level of wealth increases.

4.2. A linear Mean-Variance model under Just-Pope technology

The duality model is developed here under the following assumptions: linear mean-variance risk preferences or constant absolute risk aversion (CARA), Just-Pope technology, and price

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[11] Note that a can also be used to compare the risk aversion of two agents. If a_a and a_b are the coefficients derived from the vNM utility functions U_a and U_b, respectively, then agent a is more risk averse than agent b if $a_a \geq a_b$.

certainty. These assumptions regarding preferences and technology albeit restrictive, have often been employed in empirical research in agriculture (Chavas and Pope 1982; Just and Pope 1978). Moreover these assumptions imply that the farm's objective function is almost linear in parameters, which simplifies exposition of the dual approach and, to some extent, simplifies empirical application.

The farmers risk preferences are specified in terms of a utility function that is linear in expected profits $\bar{p}$ and profit variance s_p^2, viz.

$$U(p) = \bar{p} - \frac{1}{2} a s_p^2 \tag{19}$$

where $a > 0$ is the coefficient of absolute risk aversion. Profits are,

$$p = py - wx \tag{20}$$

where p, y denote the price and level of a single output, respectively, and w, x denote vectors of farm input prices and levels, respectively. The Just-Pope production function is,

$$y = f(x) + g(x)e \tag{21}$$

where farm inputs' quantities x are deterministic and e denotes a stochastic weather variable with mean and variance a The mean $\bar{e}$ and variance s^2. of output conditional on x are;

$$\bar{y} = f(x) + g(x)\bar{e}$$

$$s_y^2 = g(x)^2 s^2$$

The mean and variance of profit conditional on farm input levels and prices are;

$$\bar{p}(x) = p\bar{y} - wx = pf(x) + pg(x)\bar{e} - wx s_p^2 = p^2 s_y^2 = p^2 g(x)^2 s^2$$

Substituting equation (21) into (19), the producer's choice problem is;

$$V(p, w, \bar{e}, s^2) = \max_{x \geq 0} pf(x) + pg(x)\bar{e} - wx - \frac{1}{2} a \; p^2 g(x)^2 s^2 \tag{22}$$

where $V(p, w, \bar{e}, s^2)$ is the producers dual indirect utility function, that is, the relation between maximum feasible utility U and exogenous variables $(p, w, \bar{e}, s^2)$. Properties of this utility function are summarized in the following proposition (Coyle, 1999);

. Assume existence of the dual utility function $V(.)$ (22) and twice differentiability. Then

Proposition 1 (Coyle, 1999),

(a) $V(lp, lw, l\bar{e}, s^2 / l) = lV(p, w, \bar{e}, s^2) \quad \mathit{for} \;\; l > 0$

(b) (i) $\bar{y} = \frac{\partial V}{\partial p} + a\, p s_y^2$

(ii) $x_i = -\frac{\partial V}{\partial p} \quad i = 1, \ldots, n$

(iii) $s_y^2 = -\frac{\partial V}{\partial s^2} \frac{2s^2}{a\, p^2}$

(c) $V(.)$ is convex in w, and more generally $\left[V(.)\right]_{zz} + \left[p^2\right]_{zz} s_y^2 (a/2)$ is symmetric positive semidefinite, where $\left[V(.)\right]_{zz}$ and $\left[p^2\right]_{zz}$ are Hessian matrices of second derivatives of $V(.)$ and p^2 with respect to $z=(p,w)$.[12]

This proposition generalizes the homogeneity, Hotelling's lemma, and convexity properties of standard risk-neutral models. Under risk neutrality, a dual profit function $p(p,w)$ is linear homogeneous and convex in (p,w) and satisfies Hotelling's lemma. Since the objective function for the producers maximization problem (22) is nonlinear in parameters $(p,w,\bar{e},s^2)$ due to the term $p^2 s_y^2 = p^2 \; g(x)^2 s^2$, the standard homogeneity and convexity properties are modified. Proposition la indicates that the dual $V(.)$ is linear homogeneous in $(p,w,\bar{e},1/s^2)$ rather than in (p,w); that is, decisions $(x,\bar{y})$ are homogeneous of degree zero in $(p,w,1/s^2)$ rather than in (p,w). Proposition 1a indicates that the dual $V(.)$ is convex in input prices w but not in (p,w).

The most important result for empirical applications is the generalization of Hotelling's lemma indicated in *proposition lb*. Equations lb(ii) relating input demands to derivatives of the dual with respect to input prices are analogous to the standard Hotelling's lemma under risk neutrality and envelope results under price risk and CARA. Equations 1b(i) provide a specification of expected output supply equation in terms of the dual: it relates output supply $\bar{y}$ to the duals derivative of $V(.)$ relative to p and the endogenous output variance s_y^2. Equations 1b(iii) relate output variance s_y^2 to a derivative of the dual relative to weather variance (s_e^2). Risk aversion and output uncertainty imply that the producer considers the impact of input choices x on output uncertainty as well as on expected output; that is, $\bar{y}$ and s_y^2 are selected by x, as is indicated in equation (22). Consequently output variance s_y^2 is a decision variable for the firm along with input levels x and expected output $\bar{y}$. The generalized Hotelling's lemma equation b(iii) specifies the firm's output variance decision s_y^2 in terms of the derivative of the dual with respect to weather variance s_e^2. This equation can be estimated jointly with factor demand and output supply equations.

This duality model endogenizes a firm's decisions regarding output uncertainty. Standard risk-neutral models and models with price risk ignore output uncertainty (Coyle, 1992). Output uncertainty has been incorporated into recent cost function models (Pope and Chavas, 1996; Pope and Just, 1998), but these models treat output uncertainty as exogenous to the model.

Proposition 1 indicates that equations for decision variables $(x,\bar{y},s_y^2)$ can be derived from a flexible functional form for the dual essentially as in static risk-neutral models. For example, models based on a normalized quadratic function for the dual can be constructed as follows. Selecting an input price w^0 as *numéraire*., in addition to the netputs normalization, we define the following normalizations:

$$V^* = \frac{V}{w^0};\; s_e^{2*} = s_e^2 w^0$$

Applying the homogeneity property in proposition 1a (for $l = 1/w^0$) to $V(p,w,\bar{e},s^2)$ yields

[12] Just and Pope (1978) suggest that model 4 implies that V is decreasing in w. Nondecreasing in $\bar{e}$, and nonincreasing in s^2 (assuming $g(x) \geq 0$ and $a \geq 0$);

$V^*(p^*, w^*, \bar{e}, s^{2*})$. Homogeneity condition implies the following form of the derivative of V^* relative to s_e^2;

$$\frac{\partial V}{\partial s_e^2} = w^0 \frac{\partial V^*}{\partial s_e^{2*}}$$

Thus, assuming a quadratic function for $V^*(.)$, proposition 1b yields

$$\bar{y} = a_1 + \sum_i a_{1i} z_i + a\, p s_y^2 \tag{23}$$

$$x_i = -a_i - \sum_j a_{ij} z_j \quad for \quad i \neq 0 \tag{24}$$

$$s_y^2 = -\left(a_m + \sum_i a_{mi} z_i \right) \frac{2 s_e^2}{a\, p^{*2}} \tag{25}$$

Where terms z_i (z_j) stand for normalized variables $\{p^*, w^*, \bar{e}, s^{2*}\}$. Estimates of the coefficient of risk aversion a can be obtained from equation (23) or (25) (by substituting s_y^2 in (23) by its expression in (25)). Risk neutrality can be tested in terms of the joint restrictions $a = 0$ and s^2 excluded from the dual in the specification of expected output supply and factor demands.

Consistent estimates of expected output supply and input demand equations lb(i) and (ii) can be obtained by standard methods (similar to risk-neutral dual models) assuming (adaptative or rational) expectations for output. In contrast, direct estimation of a Just-Pope production function often requires more complex methods (Just and Pope 1978; Saha, Havenner, and Talpaz, 1997). Thus, the standard advantages of a dual approach over a primal approach for specification of policy relations and estimation in the deterministic case (Fuss and McFadden, 1979) may be amplified for this model. On the other hand, the dual approach does require estimates of mean and variance for a weather variable e.

4.3. *A multi-stochastic-output model with land allocation*

We now develop a duality model assuming linear mean-variance risk preferences (constant Absolute Risk Aversion), a general technology with multiple stochastic outputs (rather than a Just-Pope technology), and uncertainty regarding output levels. Risk preferences are specified in terms of a mean-variance utility function $U = U(\bar{p}, s_p^2)$ where $(\bar{p}, s_p^2)$ are the mean and variance of profit which is a random variable due to the stochastic aspect of the bundle of produced quantities y_i. That is, randomness of profit is imputable entirely to the revenue component rather than cost component since both input prices and quantities are supposed deterministic.

Revenues for the multi-output firm are

$$R = p^T y \tag{26}$$

where T denotes transposition. p and y are price and quantity vectors, respectively, for m outputs. Output quantities y are uncertain, $\bar{y}$ is the vector of expected output quantities, and Ω_y is the $(m \times m)$ quantities covariance matrix. The joint multioutput stochastic technology is designated as $F(y,x,e)=0$, where stochastic output levels y are jointly determined by nonstochastic farm input levels x and stochastic weather variable (e) with mean and variance ($\bar{e}, s^2$). The mean and variance of the probability distribution for revenues R are designated as:

$$\bar{R} = p^T \bar{y},\ s_R^2 = p^T \Omega_y p$$

Making use of a linear Mean-Variance utility form (19), the producer's maximization problem is;

$$\begin{aligned} \mathrm{V}(\mathrm{p,w},\Omega_y,\bar{e},s^2) &= \max_{x\geq 0} U \\ &= \max_{x\geq 0} \bar{R}(x,p,\bar{e},M^e) - wx - \tfrac{1}{2} a\, s_R^2(x,p,\Omega_y,\bar{e},s^2,M^v) \end{aligned}$$

where (M^e, M^v) are all other moments determining R and s_R^2 respectively.

Application of the above methodology is illustrated here using the same data set for Tunisia cereal crops as in section 3. This permits a comparison with results reported there under the assumption of deterministic yields. This annual time-series data set (1963-99) of prices and quantities is aggregated national level, and such aggregate data generally understate substantially the variation or uncertainty of yields at the farm level. The model assumes three outputs (hard wheat, tender wheat and barely), two variable inputs (labor and fertilizers), one quasi-fixed input (capital services). Land is introduced into the model as acreage allowed to each crop as in section 3 in order to account for land allocation under fixed total acreage (at least in the short term). Constant returns to scale is imposed[13]. We assume that crops output quantities are uncertain.

Although production data is available only at the national level, data on monthly rainfall level are available for eighteen weather stations within Tunisia. Using data on rainfall level and data on regional shares allocated to cereal crops, we were able to construct a weighted mean cereal-specific weather variable based on weighting both monthly and regional rainfall levels using biological water value and land allocation. Mean and variance of weather at time t were calculated as a weighted past realizations;

$$\bar{e}_t = 0.5 e_{t-1} + 0.33 e_{t-2} + 0.17 e_{t-3} \tag{27}$$

$$\begin{aligned} s_{e_t}^2 &= 0.5(e_{t-1} - \bar{e}_{t-1})^2 + 0.33(e_{t-2} - \bar{e}_{t-2})^2 \\ &\quad + 0.17(e_{t-3} - \bar{e}_{t-3})^2 \end{aligned} \tag{28}$$

Mean expression fit adaptative expectation where believes at time t is a weighted average of past realizations. The current variance equals the sum of squares of prediction errors of the three previous years, with declining weights similar to various other studies (Chavas and Holt, 1990 and Coyle, 1999).

Means ($\bar{y}$) and Variances (Ω_y) of crops quantities was defined similarly to equation (27) and (28) respectively. Crops quantities' covariances (Ω_{ij}) were defined as follows;

$$\begin{aligned} \Omega_{ij_t} &= 0.5(y_{it-1} - \bar{y}_{it-1})(y_{jt-1} - \bar{y}_{jt-1}) \\ &\quad + 0.33(y_{it-2} - \bar{y}_{it-2})(y_{jt-2} - \bar{y}_{jt-2}) \\ &\quad + 0.17(y_{it-3} - \bar{y}_{it-3})(y_{jt-3} - \bar{y}_{jt-3}) \end{aligned} \tag{29}$$

[13] Construction of variables is discussed in section 3.

These proxies presumably provide poor measures of outputs' uncertainty at the farm level. However, farm-level data on output was unavailable.
Using *proposition 1*, a system of netputs supply/demand, land allocation and outputs' variances equations is derived,

$$\bar{y}_i = \frac{\partial V(.)}{\partial p_i} + a\left[\frac{\partial\left(p^T \Omega_y p\right)}{\partial p_i}\right] \qquad i = 1,2,3 \tag{30}$$

$$x_j = -\frac{\partial V(.)}{\partial w_j} \qquad j = 1,2 \tag{31}$$

$$\frac{\partial V(.)}{\partial l_i} = 0 \qquad i = 1,2,3 \tag{32}$$

$$\Omega_{ii} = -\left(\frac{2}{a}\right)\frac{\partial V(.)}{\partial s_e^2}\frac{s_e^2}{p_i^2} \qquad i = 1,2,3 \tag{33}$$

where equations (32) stand for land allocation. Note that outputs variances Ω_{ii} can be eliminated from the outputs supply equations by substituting (33) into (30). Thus, expected outputs supply relations can, in principle, be estimated independently of errors in measuring Ω_{ii}.
Accommodating linear homogeneity within the framework of a normalized Quadratic model as suggested by Coyle (1999), variable inputs and expected outputs are normalized by the fixed input's quantity (k). Outputs variance is normalized by k^2 and k is inc1uded as a separate explanatory variable.
The model specified in section 3 is used assuming a generalization of a normalized Quadratic dual profit function using labor price as *numéraire*. These equations can be modified as follows to accommodate outputs uncertainty:

$$\begin{aligned} y_i/k &= a_i + \sum\nolimits_{i=1}^{3} a_{ij} p_j + \sum\nolimits_{i=1}^{3} c_{ij} l_j + a_{i4} w_4 + b_{i1} s^2 w_5 + b_{i2} w_5 \\ &\quad + b_{i3}\bar{e} + b_{i4}k + a\sum\nolimits_{j=1}^{3} p_{j\Omega_{ij}} + a_{it}t \\ x_4/k &= -[a_4 + \sum\nolimits_{i=1}^{3} a_{ij} p_j + \sum\nolimits_{i=1}^{3} c_{4j} l_j + a_{44} w_4 + b_{41} s^2 w_5 \\ &\quad + b_{42} w_5 + b_{43}\bar{e} + b_{44}k + a_{4t}t] \\ c_i &+ \sum\nolimits_{j=7}^{9} d_{ij} l_j + \sum\nolimits_{k=1}^{3} c_{ik} p_k + c_{i4} w_4 + b_{i1} s^2 w_5 + b_{i2} w_5 + b_{i3}\bar{e} \\ &\quad + b_{i4}k + c_{it}t = 0 \\ \Omega_{ii}/k^2 &= -\left(\tfrac{2}{a}\right)\tfrac{s_e^2}{p_i^2}[e_i + \sum\nolimits_{j=1}^{3} b_{ij} p_j + b_{i4} w_4 + e_{i1} s^2 w_5 + e_{i2} w_5 \\ &\quad + e_{i3}\bar{e} + e_{i4}k + e_{it}t] \end{aligned}$$

Where t is a time trend. Outputs quantities' variances are set to be endogenous using (33).

Existence of an indirect utility function $V(.)$ consistent with equations (30)-(33) implies symmetry restrictions across netputs' supply/demand and land allocation equations;

$$a_{ij} = a_{ji}$$

$$c_{ij} = c_{ji}$$

Symmetry restrictions regarding the equation for the variances of outputs are also implemented.

4.4. Main results

A nonlinear three stage least square was used to estimate parameters of the above model. Hypotheses of unit roots using standard unit root tests (Dickey-Fuller, Phillips-Perron) were rejected for dependent and in dependent variables except for normalized price of fertilizers and this variable was stationarized accordingly. Thus, variables exhibiting trend were assumed to be trend stationary, and this case was accommodated by including time trend in regression equations.

Nonlinear three stage least square estimates are reported in the Appendix. Except for the time trend, coefficients can be interpreted as elasticities in 1999 (variables were normalized to 1 for 1999 and all across-equation restrictions were transformed accordingly).

As anticipated under risk aversion and crops' yield uncertainty, the coefficients b_{i1} (for $i = 1.2.3$) of weather variance in the crops' output equation are significant and negative. The significant negative sign for the b_{i1}'s implies that expected crops' output supply are decreasing in weather variance, as expected under risk aversion and yield uncertainty.

Outputs are also increasing in own prices. Fertilizers' demand is decreasing in own price, and increasing in outputs' prices. Fertilizers' demand is found to be decreasing in weather variance (but not to its expectation) which confirm the contention implying that fertilizers are yield variability increasing. On the other hand, estimate of the coefficient of risk aversion a were found to be significant at the 5 percent significance level. Coefficients in the equation for crops' output variances (Ω_{ii}) are insignificant implying that decisions about endowments' allocation made by producers doesn't take into account yields' variances as target variable.

The *numéraire* price w_5 is significant in the model. Thus the homogeneity restrictions under the standard risk-neutral model and under CARA are rejected. Weather uncertainty effects (proxied by weather variance calculated using equation (28)) are also highly significant. The significance of weather variance suggests that this model deals both with direct and indirect effect of random environment on outputs supply.

Table 3. Own and cross price elasticities (Mean-Variance model)

	Table 1. elasticity of netput (column) relative to the price of (row) :				
	1	**2**	**3**	**4**	**5**
1	0,726	-0,724	0,142	-1,407	0,080
2	-0,023	0,146	-1,338	-3,675	-1,713
3	0,151	-1,331	0,433	-2,385	-0,519
4	0,352	1,014	0,547	-1,224	-0,389
5	-0,047	0,780	0,291	-0,762	-0,346

Elasticities are calculated at sample mean

b_{i1}'s ($i = 7.8.9$) that depict weather variance effect on land allocation are all highly significant which implies that weather variability seems to have an effect on land allocation

between crops within a multi-output activity. While weather variability is land reducing for both hard and tender wheat, this variability increase land allocated to barely which is well known as resisting cereal to climatic variations. Results concerning price elasticities for both supply and land allocation were found to be very similar to results from the model's deterministic components implemented in the framework of section 3.
Price elasticities presented in Table 3 are partial price elasticities: they represent the intensity effect of a price change. All of the own price elasticities of the three cereals and the variable inputs have the correct sign theoretically, that is positive definiteness of the Hessian implies that all own output (input) price elasticities are positive (negative). The exception is the positive sign on the hard wheat labor application in response to an increase in the price of labor. A 1 percent increase in fertilizers price has an elastic effect for the three crops.

Table 4. effect of netput prices on land allocation (Mean-Variance model)

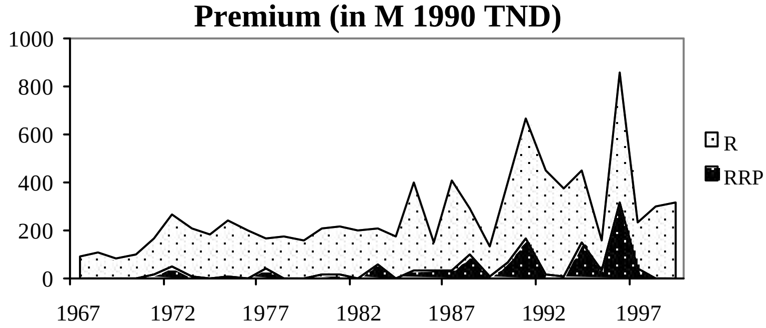
Change in land allcation relative to a 1% change in the price of :

Relative to:	Acerage 1	Acerage 2	Acerage 3
1	0,186	0,004	0,358
2	-0,006	-0,067	0,681
3	-0,191	-0,711	0,483
4	-0,501	0,349	-0,293
5	-0,160	-0,111	0,088

Elasticities are calculated at sample mean

The inclusion of the area allocation helps to capture the full complexity of the supply response (Table 4). For instance, an increase in the output prices for hard wheat and barely results in an increase in there land shares. Besides, an increase in the tender wheat's price results a decrease in its acreage allocation. This salient finding is a considerable improvement of the model considered in section .3 assuming risk neutrality.
Making use of expression (19) of the mean-variance utility function for the crops' returns (rather than the profit), we were able to calculate the risk premium relative to total returns at every sample point. The average Relative Risk Premium (RRP) during the period 1967-1999 was equal to 13.58 percent.

Figure 1. Total returns and Relative Risk Premium (in M 1990 TND)



As shown in Figure 1, risk premium has significantly increased since the early 80's. This date corresponds to the beginning of the implementation of the Agricultural Structural Adjustment Program (PASA) in Tunisia which aimed among others at the elimination of agricultural inputs' subsidization.

1. Concluding remarks

In the framework of this paper, it was question to summarize the neoclassical theory of the firm and to introduce key elements of duality theory. In section 3, it has been attempted to implement, within a dual approach, a supply/demand model based on profit function in which land allocation are set homogenous and conditional on netputs' prices. Aggregated data from Tunisia cereal crops sector are used and results was characterized by a low price elasticities of supply. In order to modelize risk non-neutrality into the implemented dual approach, mean-variance utility function of profit was introduced in section 4 to be the objective function for the producer's optimization process rather than profit. Uncertainty was set as consequence of a weather variable variance and randomness of outputs' quantities were set endogenous and conditional on weather variable variance as suggested by Just and Pope (1978) and Coyle (1999). The main innovation in this model was the implementation of a multi-stochastic-output function which implies the consideration of covariances between random output quantities.

Weather variable variance was found to have a negative impact on expected supplied quantity for both three crops but its impact was found to be non significant on input demand and land allocation. Estimate of Absolute Risk Aversion coefficient was highly significant and had the right theoretical sign which is the main strong point of the model comparing to previous works that failed to prove significant Absolute Risk Aversion coefficient. Nevertheless, insignificance of the coefficients in the output variance Ω_{ii} equation suggests that the use of aggregated measures of variance of crop output employed here are poor proxies for yield uncertainty at the farm level.

Risk Premium for the Tunisian cereal crops sector has considerably increased since the 80's. this increase has logically several roots such as the great rainfall weather variability during this period and an increase in the utilization of Fertilizers. Liberalization process undertaken by the Tunisian government since the mid 80's which had as objective the progressive elimination of agricultural inputs' subsidization ought to have a part of responsibility.

In sum, although a set of good results, the model isn't free of several shortcomings. Using aggregated data rather than farm-level data is not the best information for risk assessment and ought to lead to number of criticisms. In next works, farm-level data from farm-household producers in low rainfall locations in Tunisia ought to be used in order to assess production technology under uncertainty. Productivity and efficiency issues under uncertainty ought to be implemented.

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Appendix

Table A1. Parameter Estimates : model under risk neutrality

Coefficient	Estimate	T-Ratio	Coefficient	Estimate	T-Ratio
a_1	0.970375	2.912230	b_{32}	-0.209789	-2.524269
a_{11}	0.765141	2.024976	b_{33}	0.163033	1.780729
a_{12}	-0.008285	-0.091506	a_{34}	0.604802	4.847827
a_{1k}	0.084606	3.614691	a_4	1.428938	3.563699
b_{11}	0.422752	8.500786	a_{44}	-0.072779	-0.193492
b_{12}	-0.416628	-2.410764	a_{4k}	0.405478	13.25700
b_{13}	-0.579188	-2.682655	b_{41}	0.145128	1.513732
a_{13}	0.231425	2.012166	b_{42}	-0.708453	-6.622462
a_{14}	-0.000918	-0.005031	b_{43}	0.351473	2.086011
a_{1t}	-0.035323	-1.824003	a_{4t}	-0.110281	-5.724179
a_2	0.384957	0.347463	b_1	-2.203365	-9.278938
a_{22}	0.293651	1.973121	b_{1k}	0.006852	0.613461
a_{2k}	0.133162	4.452740	$\mathbf{c}_{11}$	1.025543	1.978541
b_{21}	-0.217399	-5.454155	$\mathbf{c}_{22}$	1.554799	2.054877
b_{22}	-0.112389	-1.622816	$\mathbf{c}_{12}$	0.072802	0.858034
b_{23}	0.097953	1.340314	$\mathbf{c}_{13}$	-0.472172	-4.544378
a_{23}	-0.802097	-6.480139	b_{1t}	0.070624	6.751203
a_{24}	0.024446	0.208259	b_2	0.304460	1.025069
a_{2t}	0.105340	2.203124	b_{2k}	0.031563	1.470514
a_3	0.784690	6.727418	$\mathbf{c}_{23}$	-0.396059	-2.928538
a_{33}	2.174122	4.665378	b_{2t}	0.020379	1.541150
a_{3k}	-1.486167	-3.674941	b_3	0.146168	0.367561
b_{31}	-0.324074	-5.412264	b_{3k}	0.022287	1.020215
			b_{3t}	-0.032966	-2.137725

Table A2. Non linear 3sLS Parameter Estimates : model under risk aversion

Coefficient	Estimate	T-Ratio	Coefficient	Estimate	T-Ratio
a_1	0,651295	1.489045	b_{41}	0,567107	3.509519
a_{11}	0,809555	2.634036	b_{42}	0,096133	1.597043
a_{12}	-0,835619	-1.060328	b_{43}	1,037502	1.451983
a_{13}	0,169676	0.433975	b_{44}	0,00019	0.387929
a_{14}	-0,70659	-3.213760	c_7	4,397151	7.413989
c_{11}	0,731059	4.760648	b_{74}	1,087672	2.530337
c_{12}	-0,025363	-0.251520	b_{72}	0,903243	1.717751
c_{13}	-0,782555	-5.533894	b_{77}	-1,903243	-1.797487
b_{11}	-0,226909	-4.085044	d_{78}	-0,463996	-0.837039
b_{12}	-2,262167	-2.637699	d_{79}	-0,209382	-2.230407
b_{13}	0,004093	1.452289	b_{71}	0,980614	10.48933
b_{14}	0,000261	1.640992	b_{73}	-0,100295	-3.673820
a	0,0001	1,942605	c_{7t}	-0,591326	-3.287951
a_2	2,082723	3.552354	c_8	-0,059588	-3.442555
a_{22}	0,1635508	4.587336	b_{84}	-1,829101	-2.013977
a_{23}	-1,555466	-2.817587	b_{82}	3,602973	3.517060
a_{24}	-1,882667	-7.913696	b_{88}	2,81584	4.254421
c_{21}	1,210594	5.844618	d_{89}	-3,475047	-4.941802
c_{22}	0,383971	2.862784	b_{81}	0,112757	0.929767
c_{23}	-1,487856	-8.082290	b_{83}	0,024269	0.349346
b_{21}	-0,272524	-3.665988	c_{8t}	-1,575155	-3.232204
b_{22}	-2,819418	-2.537657	c_9	0,038996	2.661134
b_{23}	-0,001205	-0.631031	b_{99}	1,038996	2.497221
b_{24}	0,16681	0.206047	b_{94}	-3,526954	-6.508411
a_3	1,38459	1.861365	b_{92}	1,636911	2.596388
a_{33}	0,515329	5.583280	b_{91}	-1,30384	-2.449143
a_{34}	-1,178314	-3.863890	b_{93}	0,095734	2.377136
c_{31}	0,401511	1.621254	e_1	1,082182	3.858662
c_{32}	0,292574	1.800272	e_{13}	0,00162	0.774413
c_{33}	-1,069669	-4.432671	e_{11}	-0,000279	-0.468278
b_{31}	-0,280098	-3.018073	e_{14}	-0,003784	-0.817852
b_{22}	-5,689744	-4.412002	e_2	-0,001884	-0.646716
b_{33}	0,000058	0.114931	e_{23}	-0,00112	-0.432169
b_{34}	1,242523	1.288359	e_{21}	0,000413	0.513884
a_4	-3,626236	-6.176154	e_{24}	0,003099	0.602857
a_{44}	0,864724	2.781055	e_3	0,000231	0.065335
c_{41}	-1,005635	-6.318531	e_{33}	0,000369	0.508506
c_{42}	-1,005635	-6.318531	e_{31}	-0,000114	-0.427676
c_{43}	0,287128	2.712257	e_{34}	-0,00035	-0.223723