

Redistribution through education and other mechanisms under capital-markets imperfections and uncertainty: which is the maximizing welfare policy?

By: Ben Mimoun Mohamed*

Abstract:

Subsidies to education are frequently assess as a powerful tool for altering the income distribution especially when credits-markets are imperfect. This paper compares the education subsidy policy to other redistribution mechanisms on both inequality and economic efficiency grounds in a context of a general equilibrium model of schooling investment under uncertainty and credits-market imperfection. In the first part of the paper, education subsidies regime is compared to the negative income tax and the unskilled wage subsidy policies. We show that education subsidies are the optimal policy unless the degree of inequality aversion is not too low and financing the subsidy is not too distortive. Given that education subsidies raises the efficiency-inequality trade-off issue, we have proposed in the second part of the paper a second package of policies based on governmental education loans as a solution to limit this trade-off. We show that combining between a pure public loan and education subsidies is optimal than the solely pure public loan and when the reimbursement of this loan is made conditional to the success event. Furthermore this combination policy outperforms all other policies whatever is the degree of inequality aversion.

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* CED – TEAM, Université de Paris1-Panthéon-Sorbonne. 106-112 Bd de l'Hôpital, 75013 Paris.

E-mail : Mohamed.Benmimoun@malix.univ-paris1.fr

Introduction

Most countries subsidize, to a greater or lesser extent higher education. This policy is frequently justified by capital-market imperfections, which prevent agents from borrowing against future human capital income. Subsidies to education are therefore intended to provide equality of chances to all agents no matter what their family wealth is, and hence to improve the distribution of future incomes. Once put into distributional context, it is convenient to compare education subsidy policy with alternative ways of redistributing incomes.

The first main objective of this paper is to compare education subsidies policy with a first package of policies on both efficiency and equity grounds, -i.e., in terms of social welfare- when capital-markets are imperfect and investment in education involves uncertainty. This package includes a negative income tax where all individuals receive a lump-sum income transfer, and unskilled wage subsidy financed through an income tax levied on skilled wage labour. Obviously, these policies exert quite different effects on the distribution of skills, labour supply, efficiency and inequality. Outcomes under these policies are evaluated in two ways: in ex-ante and ex-post points of view.

While the ex-ante evaluation focuses on the opportunity distribution of skills and thus does not take into account the labour remuneration's change, the ex-post evaluation depends on the realized distribution of skills and thus on the realized labour wages.

We show that relative to the no-government intervention, all these policies may provide in ex-ante sense more equality in incomes and utilities. In ex-post sense however, both the unskilled wage subsidy and the negative income tax policies are accompanied by greater inequality. Indeed, this may arise because these two policies lead to less investment in education, which in turn -due to the presence of complementarities between skilled and unskilled labour- widens the wage gap between skilled and unskilled workers. On the other side, investment in education is at the highest level under education subsidy policy. Therefore, this policy induces the largest equalizing effect in ex-post sense since the wage ratio is at its lowest level under this policy. Thus, on the contrary to some studies stressing the reverse redistribution effect of this policy (examples include Fernandez and Rogerson [1995] and C.G.Penalosa & K.Wälde [2000]), we show that this effect may not be a problem in an ex-post evaluation as unskilled workers benefit from the tax-subsidy due the presence of complementarities.

Furthermore, beyond the efficiency loss induced by distortionary taxes, education subsidies involve extra-efficiency losses due to the high associated marginal failure rates. That's why, once we compare this policy regime to the negative income tax policy, we find that the former outperforms the latter in terms of social welfare unless the degree of inequality aversion is not too low. For low degrees of inequality aversion however, the negative income tax policy is the optimal policy.

As far as subsidies to education raise an equality-efficiency trade-off, we have asked if there is a better mechanism to finance education that removes the investment constraint imposed by imperfect capital-markets without generating important efficiency losses. This constitutes the second objective of our paper.

The proposed mechanism consists in implanting a system of education loans provided by government. Three policy alternatives of public loans are examined: 1) a scheme of pure public loan under which all individuals may borrow from the government the amount needed to finance education and pay it back once they start working, 2) a combination of pure public loan and education subsidies where subsidies are fully financed through skilled labour wage taxation, and 3) a conditional loan reimbursement where only those who successfully complete education pays back the loan they have borrowed.

The more important result emerging from these policy alternatives is that combining between a pure loan scheme and education subsidies is the optimal policy. Furthermore, once we compare between all policies explored in this paper, this combination comes out as the optimal policy regime whatever is the degree of inequality aversion.

Our paper is seen as complementary to some studies in the increasing literature on the impact of governmental subsidies to education on welfare.

Barthélémy [2000] for example, considers a model in which human capital accumulation depends on both the personal training effort and the productivity of the public education system. The latter is represented by the teacher-pupil ratio. There are two externalities: that exerted by intergenerational transfers on schooling effort and that exerted by individual efforts on the productivity of the education system. The author shows that the sign of the optimal education subsidy depends on the relative size of these two externalities. Education is subsidized unless the intergenerational externality is relatively low. Otherwise, education is taxed. This conclusion is however derived on the basis of an utilitarian welfare function, so that no consideration is given to inequality. This may be explained by the fact that the author abstracts from the credits-market hypothesis so that all individuals may enrol in education. In this context, taxing higher education for example reduces investment at this educational level and strengthens inequality. Consequently, this policy may not be optimal on welfare ground if the society is inequality averse.

EcKhard Janeba [2000], analyses government intervention in an economy that faces a widening wage gap between skilled and unskilled workers as foreign competition intensifies. He considers two policies: a negative income tax scheme and education subsidies scheme. One of his main results is that education subsidies are a useful tool for reducing inequality only if inequality aversion is quite high and financing the subsidy is not too distortive. This result is somewhat confirmed in our analysis. However, the main insufficiency of that paper is that it abstracts from capital-market imperfections and the risk associated with education investment. Furthermore, it does not show any trade-off between equity and efficiency.

E.Hanushek, Y.Leung and K.Yilmaz [2001] assess in their analysis this trade-off in comparing between education subsidy policy on one hand and a negative income tax and unskilled wage subsidy on the other hand. As in our study, they find that education subsidy proves better in ex-post point of view and it is optimal than the two other policy regimes. However, by contrast to our hypothesis, they consider that capital markets are perfect so that eligibility to invest in education depends only on the ability of individuals. Obviously, under this hypothesis, implementing any system of public loans does not make sense and therefore their analysis does not offer any way to avoid or at least to limit efficiency losses generated by the high marginal failure rates characterizing education subsidy policy.

The paper of C.G.Penalosa & K.Wälde [2000] compares some policy regimes based on public loans with the traditional education subsidy policy when individual are borrowing constrained. In their study, public loans are implemented to avoid reverse redistribution associated with education subsidies policy. They consider so because their analysis focuses only on ex-ante evaluation and thus does not take account for the complementarities between skilled and unskilled workers. In fact, reverse redistribution may completely disappear once we focus on ex-post outcomes as unskilled workers benefit from the resulting change in labour wages. In addition, their study does not explicitly explore the effect of each considered policy regime on neither the distribution of skills, nor on social welfare. Outcomes are indeed evaluated only in terms of the difference between utility derived from investing in education and not investing. So, no consideration is attributed to the efficiency effect.

The rest of the paper is organized as follow. Section I presents the model in the benchmark case of no government intervention and briefly determines schooling attendance and success rates under the regimes of education subsidy, the negative income tax and the unskilled wage subsidy. Section II provides a comparison of these policies in terms of efficiency and inequality and assesses the dominance of education subsidies in terms of social welfare for modest or high degrees of inequality aversion. Finally in section III, we introduce the public loan system under several schemes and compare the effects of each scheme with the competitive economy and the education subsidy policy.

I - The model

The model focuses on the role of public schooling and transfers in an economy where social planner cares about both economic efficiency and inequality. In the context of capital markets-imperfection, the government intervenes through multiple redistributive mechanisms. These mechanisms comprise education subsidy, unskilled wage subsidy, a negative income tax and a system of public loans. The schooling decision involves uncertainty because individuals with different abilities have different chances of successfully completing schooling. Optimal schooling decisions are then made up on the basis of the distribution of both parental inheritances and individual abilities, the schooling cost and the expected net wages resulting from alternative public policies. Individuals become skilled in the only event of successfully completing education. Unskilled workers compromise thus all those who invest but fail in completing education and those who do not invest at all.

I – 1 Description of the economy

As in Galor and Zeira [1993], a simple form of imperfection in the credits-market is assumed. The credits-market is characterized by the possibility that a borrower may not pay back his debt. Lenders can avoid such defaults by keeping track of borrowers, but such precautionary measure is costly. Hence, an individual who borrows some amount - to finance the education cost noted by f - pays an interest rate i , which covers the lender's interest rate r and the tracking cost. In this case, initial financial wealth noted by x plays an important role in attendance decisions. Furthermore, what determines whether individuals successfully complete or not their education is the ability level of each individual noted by a .

For simplicity, the size of the population is normalized to one. Inheritances are distributed on the interval $[0, \bar{x}]$ according to the density function $f(x)$ while abilities evolve in the interval $[0, 1]$ and have a distribution function of $g(a)$. The two distributions are assumed to be independent. We assume further that individual of ability a faces the fully known probability of success noted by $P_a = a$. The probability of failing is then $1 - a$. Skilled workers earn a gross wage of w_s , while unskilled workers earn a gross wage of w_u .

I-1-1 The case of no government intervention

With no government intervention, all agents maximize a utility function given by:

$$U_j(c_j, l_j) = c_j - \mathbf{d} l_j^{\mathbf{h}} \quad (1)$$

where c and l are respectively consumption and labour supply, \mathbf{d} and \mathbf{h} ($\mathbf{d}, \mathbf{h} > 0$) are parameters related to the disutility of labour, and $j = u$ (unskilled worker), s (skilled worker).

There are three individuals' groups:

a- Individuals who inherit in first period of life an amount $x < \mathbf{f}$ and decide not to invest in education: these individuals are lenders and work as unskilled in the second period of life. Their lifetime utility is given by:

$$U_u = w_u l_u + x(1+r) - \mathbf{d} l_u^{\mathbf{h}} \quad (2)$$

b- Individuals with inheritance $x < \mathbf{f}$, but invest in education are borrowers with utility:

$$U_s^b = w_s l_s + (x - \mathbf{f})(1+i) - \mathbf{d} l_s^{\mathbf{h}} \quad \text{if they successfully complete education,}$$

and

$$U_f^b = w_u l_u + (x - \mathbf{f})(1+i) - \mathbf{d} l_u^{\mathbf{h}} \quad \text{if they fail in completing education.} \quad (3)$$

Thus, these individuals have an expected utility from investing in education given by the appropriately weighted average of these two utilities:

$$EU^b = aU_s^b + (1-a)U_f^b \quad (4)$$

c- Individuals with inheritance $x \geq \mathbf{f}$, who invest in education are lenders with utility:

$$U_s^l = w_s l_s + (x - \mathbf{f})(1+r) - \mathbf{d} l_s^{\mathbf{h}} \quad \text{if they successfully complete education,}$$

and

$$U_f^l = w_u l_u + (x - \mathbf{f})(1+r) - \mathbf{d} l_u^{\mathbf{h}} \quad \text{if they fail to complete education.} \quad (5)$$

These individuals have an average expected utility from attending education given by:

$$EU^l = aU_s^l + (1-a)U_f^l \quad (6)$$

- From equations (2) and (6), lenders invest in education as far as $EU^l > U_u$. This yields a critical level of ability under which lenders can not invest⁽¹⁾. It's given by:

$$a^\bullet = \frac{f(1+r)}{(w_s l_s - w_u l_u) - d(l_s^h - l_u^h)} \quad (7)$$

- From equations (2) and (4), borrowers invest in education as long as $EU^b > U_u$. This condition yields a critical level of financial inheritance under which these individuals prefer not to acquire further education:

$$x^\bullet(a) = \frac{1}{(i-r)} \left[f(1+i) - a(w_s l_s - w_u l_u - d(l_s^h - l_u^h)) \right] \quad (8)$$

Summing up, the population who invests in education is given by lenders with ability higher than a^\bullet , and borrowers with financial inheritance larger than $x^\bullet(a)$, that is:

$$N_e = \int_{a^\bullet}^1 \int_f^{\bar{x}} f(x) g(a) dx da + \int_0^1 \int_{x^\bullet}^f f(x) g(a) dx da \quad (9)$$

The fraction of population who invests and succeeds is:

$$N_s = \int_{a^\bullet}^1 \int_f^{\bar{x}} a f(x) g(a) dx da + \int_0^1 \int_{x^\bullet}^f a f(x) g(a) dx da \quad (10)$$

Schooling failure rate is therefore:

$$N_f = N_e - N_s = \int_{a^\bullet}^1 \int_f^{\bar{x}} (1-a) f(x) g(a) dx da + \int_0^1 \int_{x^\bullet}^f (1-a) f(x) g(a) dx da \quad (11)$$

(1): On the contrary to our model, the one of Galor and Zeira [1993] assumes that all lenders may invest in education.

The population who chooses not to enrol at all is:

$$N_d = 1 - N_e = \int_0^1 \int_0^{\bar{x}} f(x) g(a) dx da + \int_0^{\bar{a}} \int_f^{\bar{x}} f(x) g(a) dx da \quad (12)$$

Thus, the unskilled population noted N_u is the sum of individuals who do not invest in education at all N_d , and those who invest but fail N_f : $N_u = N_d + N_f$.

In the absence of the government, the optimal labour supply choice, l_j , is simply a function of the wage rate:

$$l_j = \left(\frac{w_j}{hd} \right)^{\frac{1}{h-1}} \quad (13)$$

I-1-2 Determination of labour wages and the cost of education (f)

The economy produces a unique consumption good Y whose production technology requires two types of workers: skilled and unskilled. The following technology is assumed:

$$Y = \mathbf{g} \left(L_s \right)^a \left(L_u \right)^{1-a} \quad (14)$$

where $L_s = N_{sp} l_s$ is the effective units of educated labour, where N_{sp} is the proportion of successfully educated agents who participate in goods production. We assume in fact that successful students N_s are divided between those who directly participate in goods production N_{sp} , and those who are needed to teach N_t , i.e., $N_s = N_{sp} + N_t$. We will explicitly determinate N_t below.

$L_u = N_u l_u$ is the effective units of unskilled labour, i.e., the fraction of unskilled workers times the labour supply duration of this kind of labour.

With perfect competition in the labour market, wages are simply the marginal product for each type of workers:

$$\begin{cases} w_s = \mathbf{a} \mathbf{g} \left(\frac{L_u}{L_s} \right)^{1-a} \\ w_u = (1-\mathbf{a}) \mathbf{g} \left(\frac{L_s}{L_u} \right)^a \end{cases} \quad (15)$$

For the cost of education, it is assumed that it simply equals total wage bill of teachers since in most countries, the latter represents the largest element of public education expenditures. This allows us to study how school costs vary with the wage rate of skilled labour and the demand for schooling. We

suggest that in equilibrium, all teachers supply the same quantity of working time l_s as other educated workers.

Assume as mentioned above that a fraction N_t of skilled workers N_s is needed to teach. Each enrolled student requires h hours of teaching to achieve education and each teacher can only teach e students simultaneously. Thus we must get: $N_t l_s e = N_e h$. This means that total number of possible teaching hours must equal total number of required hours of training. Thus, the number of teachers demanded is: $N_t = \frac{N_e h}{e l_s}$, which is endogenous since N_e and l_s are endogenous. For each enrolled

student, the social cost of acquiring education f is thus measured by the total wage bill of teachers divided by the total number of enrolled students. That is:

$$f = \frac{N_t l_s w_s}{N_e} = \left(\frac{h}{e} \right) w_s \quad (16)$$

I-2 Public redistribution policies

In this section, education subsidies regime is compared with the negative income tax and unskilled wage subsidy policies. Each of these policies provide very different outcomes in terms of school attendance and success rates, wages, aggregate utility and social welfare.

I-2-1 Subsidizing the cost of education

With education subsidy, the government taxes labour income of all types of workers at a rate t and offers education at a subsidized tuition rate v .

The budget constraint facing the government simply equates total expenditures on education and tax payments:

$$v f N_e = t \left[w_s N_s l_s + w_u N_u l_u \right] \quad (17)$$

Under this policy, the distribution of skills is determined by the two following critical levels of ability and financial inheritance:

$$a^* = \frac{f(1-v)(1+r)}{(1-t)(w_s l_s - w_u l_u) - d(l_s^h - l_u^h)}, \text{ and} \quad (18a)$$

$$x^*(a) = \frac{1}{(i-r)} \left[f(1-v)(1+i) - a \left((1-t)(w_s l_s - w_u l_u) - d(l_s^h - l_u^h) \right) \right] \quad (18b)$$

Notice that, all individuals pay a tax, but only lenders being above a^* , and borrowers above $x^*(a)$ can afford education.

I-2-2 Income redistribution through negative income tax

Under this policy regime, all individuals receive a lump-sum transfer g , which acts as the guaranteed income of an agent with no other income. As above, this transfer is financed by levying a linear tax t on labour incomes.

Education must however be privately financed ($v = 0$).

Since the population in the economy is normalized to 1, the government budget constraint is:

$$g = t \left[w_s N_s l_s + w_u N_u l_u \right] \quad (19)$$

Hence, from an individual perspective, income is subsidized when $g > t w_j l_j$ and taxed otherwise. Because there are only two different wage rates, skilled workers pay a net tax, which is transferred to the unskilled ones.

The following critical levels of ability and parental inheritance determine the fraction of the population, which can afford education under this policy:

$$a^* = \frac{f(1+r)}{(1-t)(w_s l_s - w_u l_u) - d(l_s^h - l_u^h)}, \text{ and} \quad (20a)$$

$$x^*(a) = \frac{1}{(i-r)} \left[f(1+i) - a \left((1-t)(w_s l_s - w_u l_u) - d(l_s^h - l_u^h) \right) \right] \quad (20b)$$

I-2-3 Subsidizing the wage of unskilled workers

Under this policy, a tax t^e is levied on skilled workers, i.e., on those who successfully complete education. The unskilled workers receive on the other hand a proportional wage subsidy of t^u on their wages. This policy implies that transfers benefit directly to those who fail in completing education and those who can not enrol at all.

Notice that the cost of education f is once again entirely privately financed. The governmental budget equilibrium requires in this case the equality between total taxes and transfers so that:

$$t^e w_s N_s l_s = t^u w_u N_u l_u \quad (21)$$

Under this policy, labour supply of educated and uneducated write:

$$l_s = \left(\frac{w_s (1-t^e)}{hd} \right)^{\frac{1}{h-1}} \text{ and } l_u = \left(\frac{w_u (1+t^u)}{hd} \right)^{\frac{1}{h-1}} \quad (22)$$

The two critical levels determining the distribution of education become:

$$a^{\bullet} = \frac{f(1+r)}{(1-t^e)w_s l_s - (1+t^u)w_u l_u - d(l_s^h - l_u^h)}, \text{ and} \quad (23a)$$

$$x^{\bullet}(a) = \frac{1}{(i-r)} \left[f(1+i) - a \left(w_s l_s (1-t^e) - w_u l_u (1+t^u) - d(l_s^h - l_u^h) \right) \right] \quad (23b)$$

II- The comparison of policies' performances

We continue the comparison of different policies by looking at social welfare. We consider a social welfare function based on two components: efficiency as measured by the sum of the expected utility levels of all individuals and inequality as measured by the Gini index of the utility distribution. Precisely, we assume that the social planner has to maximize the following function:

$$W(t^e, t^u, v, g) = V^*(1 - r GINI) \quad (24)$$

where V is aggregate expected utility and $GINI$ is the Gini coefficient of the utility distribution. The parameter r ($r > 0$) indicates the degree of inequality aversion interpreted as the importance devoted to inequality in evaluating social welfare. Higher values of r reflect increasing concern for inequality. When $r=0$, welfare is equivalent to the aggregate utility.

II - 1 Measurement of aggregate utility and inequality

Aggregate utility is simply the sum of utilities of the non-attending population and successful and unsuccessful individuals among lenders and borrowers. That is,

$$V = \int_0^1 \int_0^{x^{\bullet}} U_u f(x) g(a) dx da + \int_0^{a^{\bullet}} \int_f^{\bar{x}} U_u f(x) g(a) dx da + \int_0^1 \int_{x^{\bullet}}^f [aU_s^b + (1-a)U_f^b] f(x) g(a) dx da + \int_{a^{\bullet}}^1 \int_f^{\bar{x}} [aU_s^l + (1-a)U_f^l] f(x) g(a) dx da \quad (25)$$

On the other side, our measurement for the Gini coefficient is based on utilities after taxes and transfers rather on labour incomes. The importance of computations based on utilities is that they capture the utility derived from parental inheritances as well as the disutility related to labour. Computations of this coefficient are based on the formula above:

$$GINI = \frac{1}{2n^2 \bar{U}} \sum_i \sum_j |U_i - U_j| \quad (26)$$

where: n is the number of groups of individuals, which is at the number of 5 under the three proposed policy regimes, \bar{U} is aggregate utility, U_i and U_j are mean utility of groups i and j respectively.

In an economy with education subsidy, the planner maximizes the welfare function subject to equations 13 which must now incorporate the tax rate, 15, 16, 17, 18a, 18b, 25 and 26. In an economy with a negative income tax, the planner maximizes the welfare function subject to equations 13, 15, 16, 19, 20a, 20b, 25 and 26. In the case of unskilled wage subsidy, the government maximizes social welfare subject to equations 15, 16, 21, 22, 23a, 23b, 25 and 26.

II - 2 Parameter values

Comparison among different policy regimes requires fixing a number of key parameters and the distribution forms of financial inheritances $f(x)$ and individual abilities $g(a)$.

The decision of parameter values begins with the preference side. Parameters related to the disutility of labour h and d are set to be 3 and 1/3 respectively. These values are probably too high from an empirical point of view. Yet, to the extent that we are also interested in the efficiency cost of taxation, it seems better to over rather than to underestimate the disutility-labour supply elasticity.

On the production side, productivity factor g and the share of skilled labour in production a are fixed so that the wage ratio between skilled and unskilled workers is exactly 2, (i.e., $w_s/w_u = 2$) in the benchmark situation with no government intervention. This yields $g \approx 3$ and $a \approx 0.65$.

Concerning the cost of education, we assume it equals 20 % of the skilled wage. On the credits-market side, lender and borrower rates of interest (r and i) are fixed at 5 % and 10 % respectively.

We assume that individual abilities are uniformly distributed on the interval $[0, 1]$. This distribution form choice is helpful here because the general-equilibrium effects are complicated enough and therefore numerical characterisation of policy regimes becomes somewhat easier. Finally, inheritances are normally distributed with a mean of 0.3 and a standard variation of 0.085.

In the competitive economy with no government, these conditions yield the following outcomes: only 60 % of the population invests in education among them 52 % are borrowers and 8 % are lenders. However only 43 % of investors successfully complete education, among them 37 % are borrowers and 6 % are lenders.

II - 3 The results

We begin presenting our results by the table (1) below which shows the corresponding equilibrium subsidy and transfer rates for each level of taxation and by policy regime.

In the absence of government, i.e., when the tax rate is zero, redistributive programs cannot take place. Redistributive policies require revenues raised through labour income taxation, and the level of tax rate simply indicates the size of each redistributive program. Hence, a higher tax rate (t) should imply

either a higher subsidy rate of education (v), or a higher transfer (g), or a higher subsidy rate for the unskilled wage (t'').

Table 1: Tax and equilibrium subsidy and transfer rates

	<i>Education subsidy</i>	<i>Negative income tax</i>	<i>Unskilled wage subsidy</i>
Tax rate (t')	v	g	t''
0.00	0.000	0.000	0.000
0.02	0.148	0.035	0.043
0.04	0.295	0.069	0.085
0.06	0.442	0.102	0.128
0.08	0.590	0.134	0.171
0.10	0.737	0.165	0.214

Table 2 shows how the different subsidy schemes affect very differently the pattern of attendance and success in higher education. In the benchmark case with no taxes or transfers, 43 percent of the population invests in education and successfully completes, while 17 percent invests but fails, and the remaining 40 percent does not invest at all. With greater education subsidies, individual willingness to pay education cost falls. This in turn increases the fraction of both lenders and borrowers who can afford education as is seen from the fall of both critical levels given by relations (18a) and (18b). Consequently, the non-attending population N_d decreases.

However, the more important impact of this policy is on the marginal failure rate. Indeed, while students who successfully complete education go from 43 % of the population in the no subsidy case to 50 % in the case of 10 % tax to support tuition education, the proportion attending but failing in completing education raises from 17 % to 39 %. This disproportion in the growth of success and failure rates reflects the high levels of education subsidy. At a 2 % tax rate, the subsidy represents only 14.8 % of education cost, while at a 10 % tax rate, this subsidy is 73.7 % (table 1) and the corresponding enrolment rate is now 89 % (table 2). As a result, many more individuals have a probability to fail in completing education and then fall into the lowest utility class.

The evolution of educational patterns is very different however for the other redistributive schemes. For the negative income tax and unskilled wage subsidy policies, increasing the levels of the transfer or of the subsidy decreases the schooling enrolment rate and the skilled workforce, with the decline being most dramatic for the unskilled wage subsidy program. Under this policy regime, higher unskilled wage subsidies (t'') which necessary imply higher taxation level of skilled wages are dissuasive and work against investment in education.

Table 2: *Distribution of school attendance and success by redistributive program and tax rate*

Tax rate (t)	Education subsidy			Negative income tax			Unskilled wage subsidy		
	N_s	N_f	N_d	N_s	N_f	N_d	N_s	N_f	N_d
0.00	0.43	0.17	0.40	0.43	0.17	0.40	0.43	0.17	0.40
0.02	0.45	0.21	0.34	0.43	0.16	0.41	0.42	0.15	0.43
0.04	0.46	0.25	0.29	0.42	0.16	0.42	0.40	0.13	0.47
0.06	0.48	0.29	0.23	0.42	0.15	0.43	0.38	0.10	0.52
0.08	0.49	0.34	0.17	0.41	0.14	0.45	0.35	0.07	0.58
0.10	0.50	0.39	0.11	0.41	0.13	0.46	0.30	0.03	0.67

Note: N_s , N_f and N_d are respectively successful, failure and non-attending school's rates.

In fact, the investment rate falls from 60 % in the absence of taxation to only 33 % in the case of 10 % tax rate. At this tax rate, the proportion of population who successfully completes education falls to 30 % compared to 41 % under the lump-sum transfer regime.

Labour income taxation and the resulting subsidies and transfers have also direct implications for wage distribution in the economy. Table 3 presents the relative wages for skilled to unskilled workers. As mentioned above, in the case of no government ($t = 0$), the gross wage of skilled workers is set to be twice that of unskilled workers. Education subsidies induce more people to invest in education and success, and the increased proportion of skilled workers drives down their relative wages. In the case of a 10 % tax rate, the relative wage of skilled workers falls to 1.56.

The other redistributive mechanisms, however, induce the opposite effect since they perform worse in terms of education participation and success. Once again, subsidizing the unskilled wage through skilled wage taxation leads to the most unequal gross wages distribution since the wage ratio grows to 4.1 with 10 % tax rate.

Table 3: *Relative wages of skilled and unskilled workers by redistributive program and tax rate*

Tax rate (t)	Education subsidy	Negative income tax	Unskilled wage subsidy
0.00	2.00	2.00	2.00
0.02	1.87	2.04	2.17
0.04	1.77	2.08	2.41
0.06	1.68	2.12	2.73
0.08	1.61	2.18	3.23
0.10	1.56	2.24	4.10

These outcomes, nevertheless, do not show the complete picture of the redistributive policies' effects on the economy. These effects must be in fact evaluated on the basis of aggregate welfare, that is, on aggregate utility and its distribution among the different classes of the population.

Figure 1 in the appendix provides a direct comparison of each of the redistributive scheme in terms of their effects on both efficiency as measured by the ex-ante aggregate utility and equity as measured by the (1-Gini) index of utilities for tax rates between zero and 10 %.

As shown by the figure, all policy regimes yield larger equality than the competitive economy with a zero tax rate, and this equality is increasing in the tax rate as indicated by the increase in the (1-Gini) index. Direct subsidies to unskilled workers appear as the more equalising tool of utility distribution in ex-ante point of view. The education subsidies program and then the negative income tax program exert however less equalizing effect.

Concerning aggregate utility, it is generally asserted that government intervention is harmful for economic efficiency because it distorts labour supply. This seems to be well the case under the wage subsidy and the negative income tax policies where aggregate expected utility is monotonically decreasing with tax rate.

However, on the contrary to this assertion, the figure shows that for small tax values, the education subsidy regime may provide more aggregate utility than in the absence of government intervention. But this utility gain rapidly disappears with higher taxation rate.

On the one hand, utility gains may arise under this policy regime because the resulting reduction in the cost of education generates gains in terms of interest rate for both lenders and borrowers, which are not redistributed. On the other hand, as the subsidy rate increases, the marginal failure rate increases (table 2) so that the expected impact of income taxation will decrease and the efficiency of the policy regime will decrease too.

For small tax rates, gains in terms of borrowers and lenders' interest rates outweigh both losses generated by labour supply distortions and by the increasing marginal failure rates. Consequently, the aggregate expected utility increases. For higher tax rates however, interest gains are smaller than the resulting "education-failure" inefficiency and the distortion effect so that aggregate utility decreases as depicted by figure 1.

The relevant question now is to know if these policies implemented in order to directly or indirectly redistribute incomes are really redistributive in ex-post point of view?

In fact, for the planner point of view, policies' effects evaluation should focus on the realized distribution of schooling rather on opportunities and must then take account for the realized wage distribution.

In ex-ante calculation sense, individuals evaluate their utilities resulting from alternative policy regimes on the basis of skill opportunities' distribution and thus take as given the labour wages. However, individual decisions also change the relative wage ratio (table 3) so that subsidies and transfers' rates change, and ex-post evaluation of the utility distribution should differ from that observed in ex-ante sense. Figure 2 displays the same comparison for the ex-post calculations of the utility distribution.

The figure indicates that government policies' effects on aggregate utility and its distribution are very different from that emerging in an ex-ante evaluation sense. For instance, two important points are driven from this figure.

First, the unskilled wage subsidy and the negative income tax policies cease to be redistributive as highlighted by the decreasing values in the (1-Gini) index and this decrease is reinforcing as wage subsidy or transfer rates increase. This inequality strengthening reflects the negative effects of these two policies on the attendance and success rates and consequently on the wage ratio.

Second, with education subsidy policy, more people invest and succeed in higher education than under any other regime. The resulting compression in skilled and unskilled labour wages is consequently the most important. This policy regime exerts thus the strongest equalizing utilities' effect. Hence, on the contrary to many studies stressing the reverse redistribution effect of education subsidies, we may see here that this effect is problematic only in an ex-ante evaluation. In an ex-post evaluation however, reverse redistribution may completely disappear due to the presence of complementarities between skilled and unskilled labour, which induce a compression in the wage ratio.

However, one may also see from figure (2) that education subsidy policy still raises the efficiency-equity trade-off since aggregate utility decreases with tax rates.

The results above reverse some general assertions, along which, unskilled wage subsidy is a powerful policy regime from the ex-post point of view because it targets those who have not attended or could not finish the higher education stage. On the other hand, education subsidy will improve the equalization of opportunities, and thus would seem to be a good policy from the ex-ante point of view. Surely, these stated assertions miss a general equilibrium perspective since the modified distributions of skills under these policy regimes induce different wage ratios and consequently different subsidies and transfers amounts in the ex-post sense.

The full comparison, however, of the differences between the ex-ante and the ex-post results requires going behind the summary outcomes depicted in figures 1 and 2.

Figures 3 and 4 compare respectively the distribution of ex-ante and ex-post utilities and the corresponding density of agents under alternative regimes with a 8 % tax rate taken as an example.

The density of agents represents the size of each group. Groups are at a number of five. The bottom groups in terms of utility are compromised of school failures (borrowers N_f^b and then lenders N_f^l), the middle group contains those who do not invest at all (N_d), and the top groups are made up of successfully school completers (borrowers N_s^b and then lenders N_s^l).

As has been already seen, from an ex-ante point of view, the unskilled wage subsidy program has the most equalizing effect. Indeed, it provides the largest utility level for agents who do not invest and for failing lenders on the one hand, and leads to the less utility level for the top group on the other hand.

Under education subsidies, reverse redistribution takes place since those who do not attempt education or lenders who fail are worse off relative to other policy regimes while successful borrowers are better off.

In ex-post evaluation however, after schooling decisions are made and the realised labour wages are considered, both the bottom and the middle groups are now better off under the education subsidy

policy than under any other regime. This is because it does not only subsidizes school failures, but it also brings up the unskilled wage. On the other hand, skilled workers (the top group) are worse off under this policy since tuition subsidies lower their realized labour wage. Therefore, the reverse redistribution effect emerging in ex-ante evaluation seems to disappear in the ex-post one since unskilled workers benefit also from the tax-subsidies due to the presence of complementarities between skilled and unskilled workers.

Summing up, education subsidy policy proves better after wages' outcomes are realized. It is also the superior policy regime in terms of utility equalization. Under the unskilled wage subsidy or the negative income tax policies, bottom and middle groups have lower realized utility than under education subsidy because the resulting wage distribution is more unequal. The resulting wage distribution is in fact only beneficial to the top groups, which are better off under these two policies than under the education subsidy policy as shown in figure 4.

II - 4 Welfare comparison

Comparison of alternative policies in terms of social welfare is based on the social welfare function given by the relation (24). On the equality ground, the education subsidy policy clearly outperforms both the unskilled wage subsidy and the negative income tax policies as depicted in figure 2. However, although all policies exert a negative effect on economic efficiency, the lowest effect is associated to the negative income tax policy.

Hence, for high (or modest) degrees of inequality aversion, education subsidies policy is the optimal policy. When the degree of inequality aversion is however too low, the negative income tax policy may outperform the other two policies in welfare terms. This is illustrated by figures 5a and 5b which show the evolution of social welfare with tax rates under the three policy regimes for respectively a very low degree of inequality aversion ($r = 0.1$) and a modest (or high) degree of inequality aversion ($r = 1$).

Notice that in the case of high inequality aversion, welfare tends to decrease when income tax rates are very high ($t > 0.08$ in figure 5b). This arises because of the too high inefficiency effect arising from high labour supply distortions and high schooling failure rates.

Summing up, education subsidies are important only if inequality aversion is at least modest and financing the subsidy is not too distortive.

III- Overcoming capital-market imperfections

As the previous discussion has shown, subsidizing education is the powerful tool to reduce inequality in ex-post point of view. However, this equalizing effect is at the cost of a huge loss in terms of efficiency. This raises the question of whether there is a better mechanism to finance education. A straightforward solution, which completely removes the investment constraint imposed by imperfect capital-markets without generating inefficiency is to abolish all subsidies to education and introduce a

government loan scheme. Education would be entirely privately financed, and capital market imperfections would be overcome by loans provided by the government.

Individuals who enrol in education can take up a loan and repay it once they start working. On the contrary to the previous redistributive policies, all individuals are found to be lenders when a loan system is implemented.

Three alternative ways in implementing the loan program are considered in this section: a pure loan scheme, a mixture of public loan and education subsidy, and a conditional loan reimbursement scheme.

III – 1 A pure governmental loan policy

It's assumed under this scheme that the government makes loans available through borrowing at an interest rate of zero. Any individual can then obtain a loan equals to the cost of education f that has to be fully reimbursed irrespective of whether or not this individual succeeds in completing this educational stage. The government budget is thus balanced since total loans are paid back

Clearly, this kind of policy has the merit to remove the constraint imposed by imperfect capital-markets so that eligibility to be educated will be governed by the only individual potential ability. Comparison between the expected utility from investing and not investing in higher education yields the following ability threshold under which agents choose not to invest:

$$a^* = \frac{f}{(w_s l_s - w_u l_u) - d(l_s^h - l_u^h)} \quad (27)$$

Hence, the fraction of population investing in education is now given by:

$$N_e = \int_{a^*}^1 g(a) da \quad (28)$$

The fraction of successful individuals is:

$$N_s = \int_{a^*}^1 a g(a) da \quad (29)$$

The fraction of population who fails in completing education is:

$$N_f = N_e - N_s = \int_{a^*}^1 (1-a) g(a) da \quad (30)$$

The population who chooses not to invest in education is then:

$$N_d = 1 - N_e = \int_0^{a^*} g(a) da \quad (31)$$

It follows that the unskilled population N_u contains agents who do not go to school at all N_d , and those who go to school but fail N_f : $N_u = N_d + N_f$

III – 2 Public loan and education subsidy

This policy regime has two components. First, as under the pure public loan, any individual can obtain a public loan that has to be fully paid back whether this individual becomes or not a skilled worker. In addition, education cost is subsidized at a rate of (v). Education subsidies are financed by taxing those who have successfully completed education. Unsuccessful students have not to pay this tax. They hence receive a net transfer, while successful students have to reimburse their own education cost (public loan net of the subsidy) and the subsidy received by unsuccessful students. Under this policy regime, the balanced government budget writes:

$$v f N_e = t w_s N_s l_s \quad (32)$$

Under this policy regime, only individuals with more than a^* given above may be educated:

$$a^* = \frac{f(1-v)}{(1-t)w_s l_s - w_u l_u - d(l_s^h - l_u^h)} \quad (33)$$

III – 3 Repaying public loan under success condition

On the contrary to the pure loan policy scheme, this policy consists in a loan the student receives from the government such that repayment only takes place in the event that he successfully completes his education, i.e., when he becomes skilled worker. Thus, the unskilled workers have not to pay back the loan they have received. However, all individuals are taxed on their labour incomes in order to raise revenues needed to cover the education cost of unsuccessful students.

The government balances its budget by choosing the labour income tax rate allowing to fully cover loans provided to failure students. The equilibrium tax rate must satisfy:

$$t = f N_f / (w_s N_s l_s + w_u N_u l_u) \quad (34)$$

The expected utility from investing in education is given by:

$$EU = x(1+r) + a \left[(1-t)w_s l_s - d l_s^h - f \right] + (1-a) \left[(1-t)w_u l_u - d l_u^h \right], \quad (35)$$

while the utility from not investing is equal to:

$$U_u = x(1+r) + (1-t)w_u l_u - d l_u^h. \quad (36)$$

Performing the difference between these two utility levels, and assuming that additional benefits from training exceed additional costs, i.e., $(1-t)(w_s l_s - w_u l_u) > f + d(l_s^h - l_u^h)$, one may see that $EU > U$.

Hence, under this policy regime, all individuals are incited to enrol in education irrespective of their ability levels.

The size of population investing in education is thus $N_e = 1$. Consequently, given the uncertainty feature of success, the fractions of population who succeed and fail in completing education are respectively:

$$N_s = \int_0^1 a g(a) da \quad (37)$$

and

$$N_f = \int_0^1 (1-a) g(a) da \quad (38)$$

III – 4 Efficiency and equality measurement

With a loan system, parental inheritances cease to be the factor of eligibility to education investment. Only individual abilities matter. When all individuals who borrow the loan must pay it back (under first and second regimes), efficiency as measured by total utility can be written as:

$$V = \int_0^{\bar{x}} \int_0^{\bar{x}} U_u f(x) g(a) dx da + \int_0^{\bar{x}} \int_0^{\bar{x}} [a U_s^l + (1-a) U_f^l] f(x) g(x) dx da \quad (39)$$

However, when reimbursement is making conditional to the success event, all individuals attempt education and thus total expected utility writes as:

$$V = \int_0^{\bar{x}} \int_0^{\bar{x}} [a U_s^l + (1-a) U_f^l] f(x) g(x) dx da \quad (40)$$

Concerning inequality, it is as above calculated by the following Gini index:

$$GINI = \frac{1}{2n^2 \bar{U}} \sum_i \sum_j |U_i - U_j| \quad (41)$$

where: n is the number of groups of individuals and equal to 3 under the first two schemes and 2 under the third one. \bar{U} is aggregate utility, U_i and U_j are mean utility of groups i and j respectively.

Under the pure public loan, the planner maximizes the welfare function subject to equations 13, 15, 16, 27, 39 and 41. Under both education subsidies and public loan, the planner maximizes welfare subject to equations 13 which must incorporate the tax rate on skilled wage, 15, 16, 32, 33, 39 and 41. In the case of conditional loan reimbursement, the government maximizes social welfare subject to equations 13 incorporating tax rate on both skilled and unskilled wages, 15, 16, 34, 40 and 41.

III – 5 The results

Table (4) below summarizes our ex-post simulations' results in terms of schooling attendance and success under each of the three schemes proposed above and the corresponding aggregate expected utility, the Gini index and the social welfare level for low and modest degree of inequality aversion.

Table 4: Summarized outcomes under different public loan schemes

			N_s	N_f	N_d	AEU	GINI	Welfare $r=0.1$	Welfare $r=1$
No government intervention			0.43	0.17	0.40	1.254	0.249	1.223	0.941
Pure public loan			0.45	0.22	0.33	1.261	0.214	1.234	0.991
Pure public loan and Education subsidy	$t = 0.00$	$v = 0.00$	0.45	0.22	0.33	1.261	0.214	1.234	0.991
	$t = 0.02$	$v = 0.09$	0.46	0.23	0.31	1.260	0.202	1.235	1.005
	$t = 0.04$	$v = 0.19$	0.46	0.25	0.29	1.259	0.189	1.235	1.020
	$t = 0.06$	$v = 0.28$	0.47	0.26	0.27	1.256	0.177	1.234	1.033
	$t = 0.08$	$v = 0.37$	0.47	0.28	0.25	1.253	0.165	1.232	1.045
	$t = 0.10$	$v = 0.47$	0.48	0.30	0.22	1.248	0.153	1.228	1.056
Public loan under conditional reimbursement ($t = 0.208$)			0.5	0.5	0.0	0.974	0.112	0.963	0.865

One may see that relative to the no government intervention case, the pure loan scheme induces only 7 % extra-schooling investment. This arises because this policy regime does not provide insurance for schooling failure since the borrowed loan must be repaid back irrespective whether investors succeed or not. This in turn discourages individuals of very low ability to obtain a loan.

Efficiency as measured by the aggregate expected utility is however higher under the pure public loan than in the competitive economy. This simply arises because schooling investment is increased without any labour distortive tax ($t = 0$).

In ex-post sense, increasing in the skilled workforce drives down the wage ratio of skilled to unskilled workers and the distribution of utilities becomes more equal as indicated by the lower level of Gini index relative to that in the competitive economy. As both efficiency and equality improve under the pure loan policy, welfare is higher under this policy than under the competitive one regardless of the degree of inequality aversion.

Outcomes are much better when pure public loan is combined with tuition subsidies. Indeed, with a 10 % tax rate for example, 78 % of the population invest now in education, from which, 48 % successfully complete. While this policy regime sharply reduces inequality (Gini index decreases), it induces also some efficiency loss because of the distortionary effect of taxation. Whatever is the level of inequality aversion however, this policy outperforms the only pure loan scheme on social welfare ground.

Under conditional public loan reimbursement, loans are paid back in the event of success only. This provides insurance against failure and all individuals borrow the loan and invest in education. The population is then equally divided between skilled and unskilled workers. The equilibrium income tax rate which allows to exactly cover the education cost of unskilled equals 20.8 %. Because of this high tax rate, aggregate expected utility is found to be lower than that observed under alternative policies. In the other hand, inequality is at a very low level (0.112) and this is due to the wage ratio compression resulting from the huge extra-success rate.

However, the conditional public loan policy provides less welfare compared to the pure loan and education subsidy policy whatever is the degree of inequality aversion. Summing up, combining between public loan and education subsidy is the optimal policy among that proposed in table 4.

Moreover, beyond policies based on loans system, this combination performs also better than both the education subsidy and the negative income tax policies explored in the previous section.

Figure 6 in the appendix summarizes the evolution of social welfare with tax rates under both the combination policy of public loan and education subsidy on one hand, and the education subsidy and the negative income tax policies on the other hand. Discontinuous lines represent welfare under low degree of inequality aversion ($r = 0.1$), while continuous ones refer to welfare under high degree of inequality aversion ($r = 1$).

As shown in section (2), the education subsidy policy outperforms the negative income tax in terms of welfare provided that the degree of inequality aversion is not too low. When the loan system is taken into account, figure (6) shows that the optimal policy among all the explored ones is a combination between a pure public loan program and education subsidies policy irrespective of the degree of inequality aversion.

Conclusions:

The main purpose of this paper was to analyse government intervention effects on social welfare in an economy where capital-markets are imperfect and investment in education involves uncertainty. Welfare is evaluated on both efficiency and equality grounds. Several redistribution policies are considered: education subsidies, a negative income tax, unskilled wage subsidy, and finally three alternative schemes based on education public loan system.

The important insight from this analysis is that these policies have quite different effects on the proportion of individuals who become skilled workers, efficiency, inequality and consequently social welfare. In particular, on the contrary to the general assertion, both direct subsidizes to unskilled wage and the negative income tax reduce inequality only from an ex-ante point of view. In the ex-post sense however, when we take account for the resulting wages' modification, these two policies are found to exert the opposite effect on income inequality. On the contrary to many studies evoking the reverse redistribution effect of education subsidies, we show that this effect may disappear in the ex-post evaluation due to the resulting wage compression effect. Furthermore, while education subsidies lead to more equal utility distribution in ex-post sense, they induce a huge loss in terms of efficiency because of the important accompanied marginal failure rate. Therefore, this policy regime is optimal unless the degree of inequality aversion is at least modest and the subsidy is not too distorsive. For very low degrees of inequality aversion however, the negative income tax policy becomes the optimal one as efficiency loss is relatively limited.

When a public loan policy is implemented, the distribution of skills depends only on the distribution of abilities.

Relative to the competitive economy, a pure public loan leads to higher efficiency since it improves enrolment rates without any distorsive labour tax. However, this improvement is not too consequent as this policy does not provide insurance against failure. That's why we have considered two alternative policies: a combination policy consisting in both loans provided by the government and education subsidies where these subsidizes are fully financed by skilled labour taxation, and a conditional loan reimbursement policy under which only successful students have to pay back the public loan they have borrowed.

We have shown that despite of the fact that all individuals may enrol in education under the conditional loan reimbursement regime, social welfare is higher when the combination between public loan and education subsidies is considered. Furthermore, this combination performs also much better if only education is subsidized or under the negative income tax whatever is the degree of inequality aversion.

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Appendix

Figure 1: Ex-ante AEU and (1-Gini) index

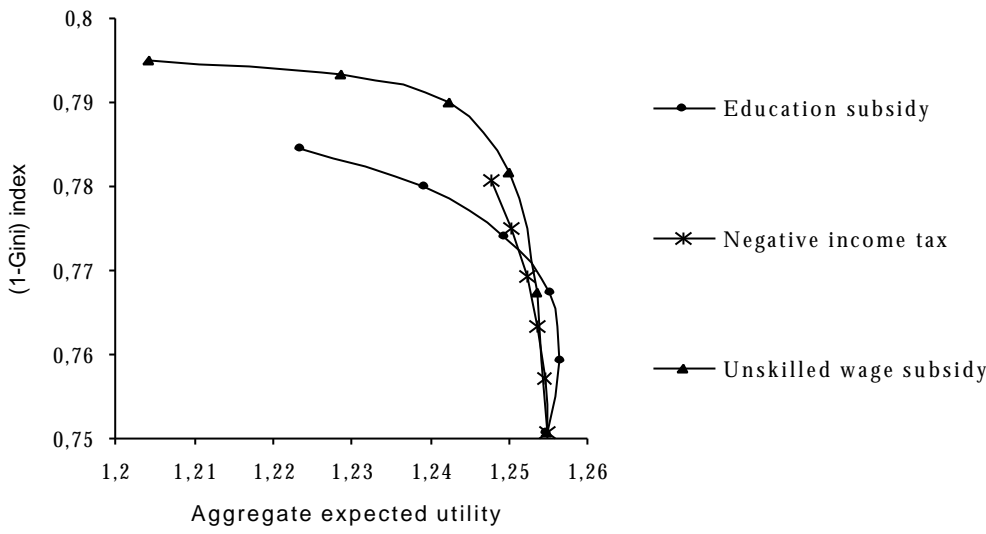


Figure 2: Ex-post AEU and (1-Gini) index

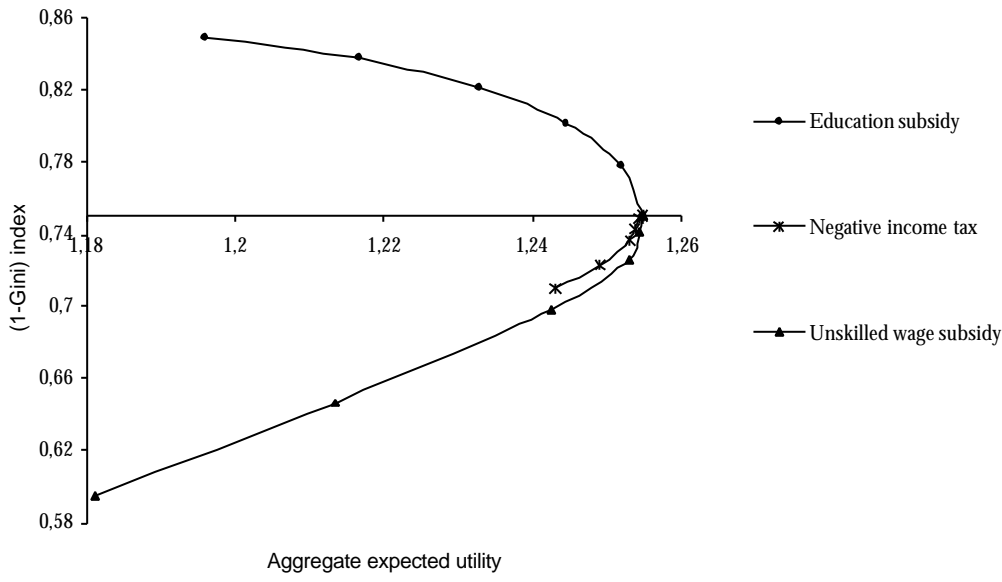


Figure 3: Ex-ante distribution of utilities

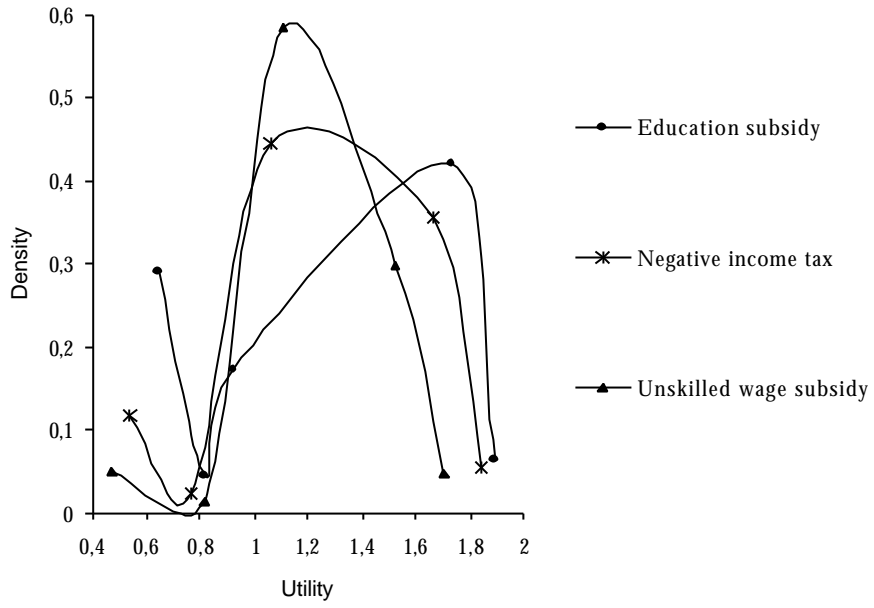


Figure 4: Ex-post distribution of utilities

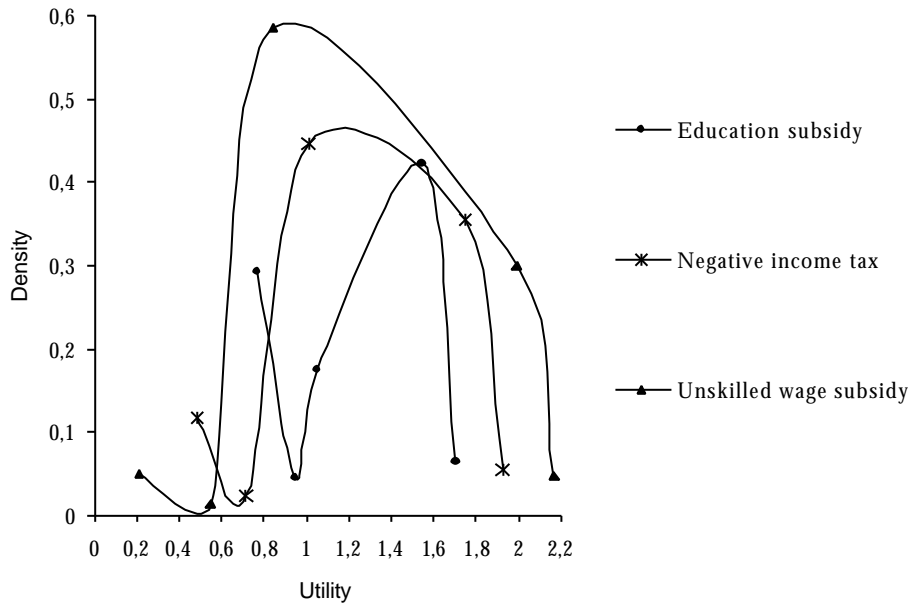


Figure 5 a: Social welfare and tax rate : $r = 0.1$

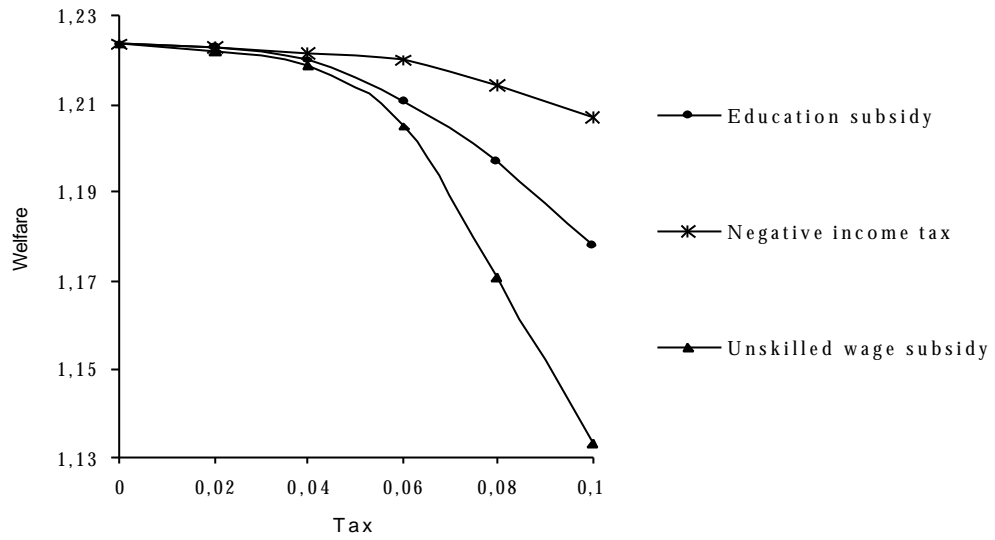


Figure 5 b: Social welfare and tax rate : $r = 1$

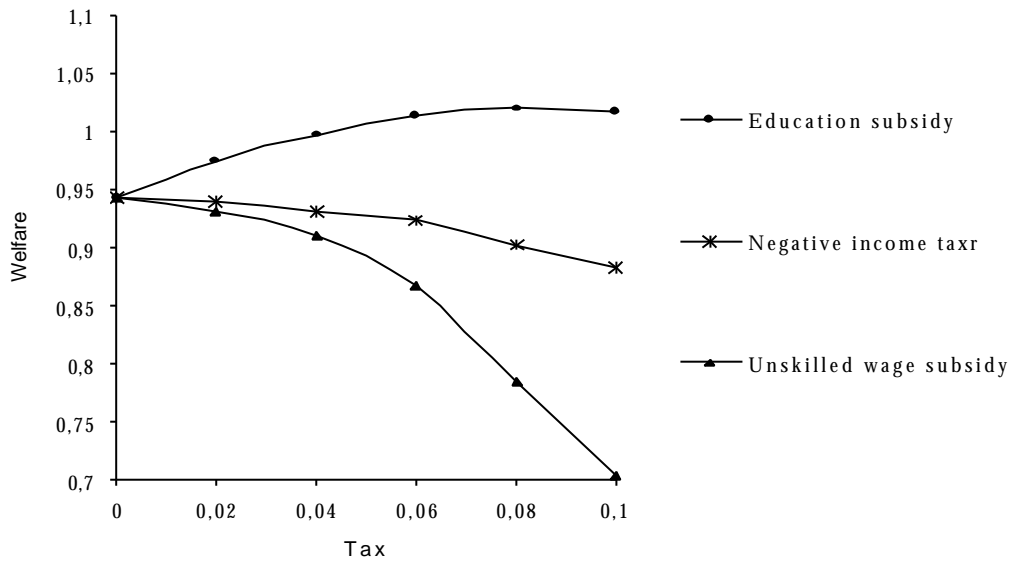


Figure 6: Social welfare and tax

